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The effect of endpoint knowledge on dot enumeration

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THE EFFECT OF ENDPOINT KNOWLEDGE ON DOT ENUMERATION

by

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ABSTRACT

The Effect of Endpoint Knowledge on Dot Enumeration

by

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This study attempts to extend the principle tenets of the Overlapping Waves Theory (Siegler, 1996), a framework designed to explain the progression of trends in cognitive development, to adult participants' performance in a dot enumeration task. Literature in the 0-100 number line estimation task (Siegler & Booth, 2004, Ashcraft & Moore, 2011) has revealed a pervasive trend in child estimation such that young children (especially those in kindergarten) respond with a logarithmic line of best fit, while children at the third grade and above overwhelmingly respond with linear estimates to this same range of numbers. A similar developmental trend is found with older children in the 0-1000 range as well (Siegler & Booth, 2004). It is argued in this work that the expression of two distinct representations, as seen in developmental number line estimation studies, is also possible in adult samples. Additionally, it is argued that the expression of one representation over another is dependent on the same cognitive components acquired in the development of a linear number sense in children. More specifically the expression of these representations is a function of the available amount of numerical information pertaining to the origin, endpoint, and subsequently the midpoint of the parameter tested.

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CHAPTER 1

INTRODUCTION

Imagine travelling to an unfamiliar city to attend a conference. You would like to arrive at the conference hotel at 8am to see a speaker, but you aren't sure what time to leave to arrive to the conference on time. You think you are about 5 miles away, so you must take a cab. Judging by the morning traffic of your home city, you anticipate a 20 minute commute to the conference hotel. This scenario incorporates many common estimates that are accomplished on a regular basis: distance, time and traffic congestion. These estimates lead to the conclusion that you should call a cab at 7:30 to arrive on time. The first time you make this estimate in a new city, you might miscalculate by 15 minutes or so, but the more experience and knowledge you acquire about this new city, the better able you will be to ground your estimates from your experiences. The concept of gaining experience or knowledge to make increasingly accurate estimates about the task at hand is crucial for efficient navigation in this specific example, although this notion extends to virtually any scenario requiring the estimation process. Without the practiced and principled effort to better approximate an accurate response through experience and an increasing knowledge base of relevant information, the output of the estimation procedure would be no better than guesswork.

Thus, the central focus of this thesis: to determine the cognitive components necessary to fully calibrate, or hone in, estimation abilities to provide estimates that closely approximate the objective value of that which is estimated. More specifically, the purpose of this study is to better understand the progression of numerical understanding of children, as examined in developmental number line estimation studies, by attempting to recreate a similar progression in an adult sample. If a comparable progression can be found in an adult sample, and the cognitive components of that trajectory can be manipulated to elicit differing patterns of performance, then these results would speak directly to the critical components achieved in development to arrive at adult-like, proficient estimation.

Estimation is a process by which people attempt to better understand the characteristics of their environment while being devoid of the necessary information (or motivation) to accurately calculate that which is unknown. The process itself is typically fast and can be applied to many situations encountered throughout day-to-day life. In review of the areas of cognition and perception that attempt to measure estimation proficiency, the general conclusion can been drawn that although the process can be rapid, people can vary wildly in terms of accuracy contingent upon their ability, experience, or on the type of task tested. Here I will examine the task designs and characteristics of the relevant studies in both developmental and adult research to highlight the crucial properties of estimation that lead to the seemingly variable performance observed in the literature.

First, the experimental designs of the heavily influential work conducted by Siegler and associates will be examined. Siegler's work with developmental populations has spurred much work into the investigation of number line estimation, and has even spawned educational interventions for the acquisition of age appropriate mappings of numbers and magnitudes in children (Siegler & Ramani, 2009). The number line estimation task referred to here is simply a horizontal line, typically with a marked origin of 0 and an endpoint of 100 or 1000. Two formats of number line estimation are common in the literature. The position to number task requires participants to estimate the numerical value of a position on the number line, denoted by a vertical hatch mark (see Ashcraft & Moore, 2011). For example, on a 0 to 100 number line, if the hatch mark was slightly to the right of the midpoint, estimates might include values near 60. The number to position task requires the participant to mark the location of a numerical value placed above the number line. For example, if presented with the number 50 for a 0 to 100 number line, adults would make a mark near the midpoint. The implications of the results from these tasks are many, and will be spelled out throughout this work to help substantiate the impetus of this thesis. Before I begin to assess the theory drawn from this area of study, it is important to clarify the major implications of the different number representations found within the literature. These representations are the crux of Siegler's theory and, by extension, the basis for the theoretical stance of the current work.

A major distinction that needs to be drawn is between the linear and logarithmic functions that are statistically fit to estimation performance (see *Figure 1*). The majority of these data come from work conducted within developmental studies (e.g., Ashcraft & Moore, 2011; Booth & Siegler, 2006, Siegler & Opfer, 2003), and these mathematical fits to the data are believed to correspond to actual mental representations of number. The linear representation is characterized by data that lie close to the x = y line of best fit (i.e., a diagonal line with a slope of 1, increasing from left to right), and signify that for each linear increase in stimulus magnitude, the result is a linear increase in the estimate of that stimulus. In other words, if numbers were to be plotted on a ruler for example, then they would increase with equal intervals between the previous and subsequent numbers, regardless of the magnitude of the stimuli.

The logarithmic representation does not assume the same pattern of correspondence between stimuli and estimates. Instead, the logarithmic representation is characterized by unequal intervals of magnitude between estimates of stimuli. The ruler plot of these values would show larger intervals between magnitudes at the lower end of the number spectrum, with decreasing intervals as the magnitude of the numbers increases. The logarithmic function is thought to mirror the compressed number representation found in animal and developmental studies, and the default structure of number representation in humans and other animals (Dehaene, 1997; Geary, Hoard, Byrd-Craven, Nugent & Numtee, 2007). The logarithmic representation has even been claimed to be an innate mental representation as it is the only representation found in animals and pre-school children (Dehaene, 1997). It is important to note here that the indication of a best line of fit is typically derived from that child's obtained R^2 coefficient as calculated through curve estimation regression for both linear and logarithmic regressions. To find the best line of fit for group data, it is common to record each child's R^2 value and compare the two representational fits through the use of a t-test (Siegler & Booth, 2004). After performing a logarithmic transformation of the data, the slope of the regression equation of transformed values has also been used to indicate the linearity or logarithmic trend of the data (Dehaene, Izard, Spelke & Pica, 2008; Izard & Dehaene, 2008).

The theory that is the basis of this argument relies on the finding that these representations of number have been shown to correspond to level of math experience within children (Ashcraft & Moore, 2011; Booth & Siegler, 2006). That is, the less experience a child has with numbers and their relations, the more likely that child will represent the number continuum with a best line of fit that is logarithmic in nature. The more experience the child acquires (typically found with increasing age and level in a school system), the more likely that child will show data that are best fit by the x = y linear representation. This pattern is found in both 0-100 and 0-1000 number line estimation (Ashcraft & Moore, 2011; Siegler & Booth, 2004; Siegler & Ramani, 2009).

Siegler and associates (Booth & Siegler, 2006; Siegler & Opfer, 2003) have consistently found a logarithmic-to-linear shift of number representation in children. With regard to the 0-100 number line, Kindergarteners and some 1st graders respond to the 0-100 task in a logarithmic fashion, while children at the third grade and above respond linearly (Ashcraft & Moore, 2011; Booth & Siegler, 2006; Siegler & Booth, 2004). In terms of the 0-1000 number line, a similar developmental progression is found. Here, second graders respond in a pattern consistent with the logarithmic representation, while sixth graders respond in a linear fashion. Third, fourth, and fifth graders respond with a mixture of linear and logarithmic patterns, with more linear responding with increasing age (Siegler & Opfer, 2003).

In the studies mentioned above, subsets of children (especially in 2nd and 3rd grades) who show a linear representation on the 0-100 task concurrently demonstrate a logarithmic representation when tested under the 0-1000 estimation task. These data not only show that there is a developmental progression from a curvilinear numerical representation to a more linear one, but also that these two representations can be expressed by the same participant, dependent on the scale or parameter tested. Such results prompted Siegler to extend the Overlapping Waves Theory (Siegler, 1996), a framework describing the cognitive development of children, to number line estimation. This framework describes strategy use development in children to be a process of skill acquisition and competition. With increasing experience within any given problem, children will acquire many strategies to complete the task, and these strategies will compete for use throughout development. It is important to note that this competition in strategy use is thought to occur to varying degrees throughout the entirety of development, as cognitive development will allow for the use of more advanced strategies to solve the problem. (Chen & Siegler, 2003).

Within the Overlapping Waves Theory, five component processes are outlined as aiding in the development of more advanced strategies (Chen & Siegler, 2003):

1) Acquiring the strategy of interest (through direct teaching, drawing analogies from one problem to another, observation of new strategy)
2) Mapping the strategy onto novel problems
3) Strengthening the strategy so that it is used consistently within given types of problems where it has begun to be used
4) Refining choices among alternative strategies or alternative forms of a single strategy
5) Executing the strategy of interest with increased efficiency

Part of this framework includes the notion that although a new strategy has been learned, the frequency and efficiency of the use of the new strategy is not 100% upon onset of learning. It is still possible for participants to fall back into using a less efficient, yet more familiar strategy. The framework makes no mention of forgetting a previously used or understood strategy. Note also that this framework has not been extended to adult estimation performance.

The current study follows the assumption that the strategy use stated in the Overlapping Waves Theory actually directs the child to respond with one representation or another, and that the strategy chosen corresponds to the cognitive ability of the child, or the amount of information made available to adults. For example, one possible strategy used by children in number line estimation could be to estimate values on the basis of the hatch mark's distance from the origin alone. Another might be to ground estimates by evaluating the hatch mark's distance from the origin and the endpoint, thus producing more accurate estimates due to an augmented base of numerical information that could be integrated into each estimate. In addition to these two strategies, a third strategy could be to use both endpoints to calculate the midpoint of the number line in order to evaluate estimates on three total reference points as opposed to just one or two. In fact, these strategies have been proposed in the literature (Ashcraft & Moore, 2011), and seem to reflect the principles outlined within the Overlapping Waves Theory above. These strategies were found to correspond with increasing number knowledge held by the child and will be revisited later in this work.

If the Overlapping Waves Theory is an accurate account of the developmental progression observed in estimation, and if it extends to adult performance, then it should be possible to determine the factors that promote the use of the linear or logarithmic representation of number. Here, studies in which these representations are found in adult performance will be examined to substantiate the central claim of this thesis: that the saliency of the parameter (the total distribution of known numbers as determined by the lower and upper bounds of the range of numbers tested) is the main determinant (or factor) in the expression of either representation in adults. The saliency of the parameter refers to how readily apparent the parameter is to the participant (e.g., explicitly labeled, abstractly represented, or conceptually salient). In developmental number line estimation studies, the parameter is always present, i.e., the continuum is presented on each trial with explicitly labeled endpoints. The logarithmic and linear representations are the product of the child's ability to extract and integrate numerical information from the image presented for each estimate. This ability has been shown to increase in proficiency with increased mathematics knowledge. For adults, this thesis assumes that the linear and logarithmic representations are a product of the amount of numerical information that is available for estimate evaluation, as adults already have the requisite number knowledge for sophisticated estimation. The next section will present instances in the literature when varied amounts of information lead to the expression of both representations. This, then, sets the stage for a study that attempts to mirror child performance by providing limited amounts of information to adults, influencing the use of the child-like understanding of number.

The basic logic of the above claim begins with the developmental data explained above. Siegler and colleagues (Ashcraft & Moore, 2011; Booth & Siegler, 2006; Siegler & Opfer, 2003) show that with increasing age, children rely less on the logarithmic representation and start developing the more linear, adult-like representation. With more information about number relations, the more linear children's estimates become, but this knowledge is context dependent in the sense of the parameter tested. This is shown on the 0-100 number line. Kindergarteners respond logarithmically, but by the second or third grade, children have fully integrated the linear representation into estimation. Although many children in the second grade respond linearly on the 0-100 task, a subset of the same children demonstrates logarithmic responding on the 0-1000 task. This occurs because children in the second grade do not have a firm understanding of this specific parameter. They do not fully understand the magnitude of the 0-1000 distribution of numbers, due to lack of experience manipulating these numbers. These results lead to the conclusion that, at least in children, expression of one representation over the other is context dependent such that a familiar and well understood parameter (e.g. 0-100) elicits linear responding, while an unfamiliar or unknown parameter (e.g. 0-1000) elicits a logarithmic response pattern.

As an example, Ashcraft and Moore (2011) found that all 20 of the third graders tested showed linear responding on the 0-100 lines, and yet 4 of these children (20%) responded logarithmically on the 0-1000 lines; similar percentages of fourth and fifth graders also responded logarithmically to the 0-1000 task. The only difference between these two tasks is the endpoint used to define the parameter of the line. The underlying principle common to both tasks is the relation of equidistant intervals between each number in the distribution of magnitudes. Adults respond linearly on both tasks because they have sufficient experience with the number relations of both ranges and are able to recognize the underlying principle of each range of numbers.

It is proposed here that children demonstrate linear responding within some ranges and logarithmic responding in others because the lack of knowledge/familiarity of the endpoint influences the child to disregard the learned rule of equal interval distances between numbers, a previously acquired skill for a smaller range of number. Instead they adopt or fall back on the analog, or less advanced, logarithmic representation. This speaks to the child's failure to map learned strategies onto novel problems, as the

Overlapping Waves Theory outlines. For third graders, the 100 endpoint is salient (meaning sufficient practice and exposure to this range of numbers has allowed them to fully understand the properties of that endpoint), but the unfamiliar endpoint of 1000 is not, due to lack of practice or understanding of this range of numbers. In order to complete the task, the child must rely on a sense of magnitude that is most salient—the logarithmic representation. In adults the expression of one representation over another is thought to be a product of the amount of information that can be extracted from a situation.

This idea of context dependency is demonstrated in a cross cultural study investigating number sense in adults as well. In a study conducted by Dehaene, Izard, Spelke, and Pica (2008), participants consisted of two distinct cultures: the Amazonian Mundurucú tribe and American adults. The tasks consisted of dot array estimation in the ranges of 1-10 and 10-100, tone enumeration in the range of 1-10 tones, and verbal number recognition in the range of 1-10 for the first and second language. For all five tasks tested, a continuum was presented on a computer screen with a sliding cursor that the participants used to indicate the magnitude of the stimuli presented. The leftmost and rightmost ends of the continuum were not labeled with numbers, as is typically the case, but instead with dot arrays, which represented the origin and endpoint of each range of numbers tested. The participants were not told or shown the magnitude of the origins or endpoints of the continua.

In the 1-10 and 10-100 dot array estimation tasks, participants were shown target dot arrays that corresponded to an exact number of dots within the range tested. The task of the participant was to slide the cursor along the continuum and place it at the position that would best describe the magnitude of the target array presented. In the 1-10 tone enumeration task, participants were provided with the same continuum/cursor design on the computer screen, with arrays representing the origin and endpoint. The participants were expected to slide the cursor to the position best representative of the number of tones they heard. Tones were presented in rapid succession, up to 10 tones in one trial. Finally, in the 1-10 verbal number recognition task, the experimenter spoke number words to the participant in two conditions (the first and second languages of the participant). The task of the participant was to slide the cursor along the continuum and place the slider at the point that would best demonstrate the number word spoken by the experimenter. The second language tested for the Mundurucú was Portuguese while the second language tested for the Americans was Spanish.

The Mundurucú's results in all five conditions were best fit by the logarithmic pattern. This held true regardless of limited education, age, or amount of second language experience received within the sample. It is important to note that although the first language of the Mundurucú can represent all numbers from 0-10, it is not customary for these people to learn and use these words. American adults performed linearly on the 1-10 array estimation task, and the two spoken language conditions, but showed logarithmic responding on both the 10-100 array estimation and tone numerosity tasks (determined through significant log-transformed regression slope coefficient). The conclusion was drawn that the Mundurucú people responded logarithmically because of their lack of number education and experience, because it was assumed that experience and education is necessary to form linear mappings of number relations.

Dehaene et al.'s (2008) cross cultural research raises important questions regarding the cause for one representation to be expressed in some situations, but not others. It seems that the logarithmic and linear representations are the product of two distinct, but flexible, mental number systems. To explore this possibility, the American performance must be examined more closely, as they exhibited both forms of representations in this study.

The cultural aspects of this study are compelling; an unschooled tribe of adults, with no habitual practice of count and number words in their language, showed logarithmic patterns typical of young children. To support the claim of parameter saliency influencing expression of representation, the current study will focus primarily on the performance of the American participants. In typical number line estimation tasks (0-100 and 0-1000 number line estimation), American adults demonstrate robust effects of expressing a linear representation. The typical task requires participants to examine the number line, view the parameters of the line, and make estimates either by providing the number that represents the mark that has already been placed on the line (position to number), or by placing a mark on the line at the location of a number that has been placed above the line (number to position). In the typical number line estimation task, the participants are provided with explicit knowledge of the parameters. The number 0 is placed on the leftmost position of the number line and the upper bound, either 100 or 1000, is labeled as well. Also, the task requires the participant to evaluate the exact position of the target stimuli (either the number presented above the line or the hatch mark previously placed on the line). In the Dehaene task (Dehaene et al., 2008), however, participants were provided with "approximate" parameters. That is, the parameters of the distribution were labeled with arrays of dots which represented the leftmost and rightmost positions of the continuum; generally, participants could not count the number of dots in the arrays, so they could only estimate the magnitude. Each target stimulus was represented in approximate terms as well; each target was an array of dots that corresponded to an exact position on the continuum, although the exact magnitude of each target was unknown to the participant. This task yielded logarithmic responding in American adults, suggesting the use of the logarithmic representation.

Beyond the perceptual dissimilarities of the two estimation tasks, a central conceptual component is present in the number line estimation task that is not present in the array estimation task designed by Dehaene: exact knowledge of the parameters and target stimuli. It seems, then, that the expression of the logarithmic representation is due to the lack of math information participants can integrate into each judgment; that is, the task requires estimation of an unknown magnitude within the bounds of a number continuum that is not salient for the participants. Without the explicit knowledge of both the parameter and magnitude of the test arrays, reliance upon the analog representation is the only way to complete this task.

Thus far, the literature discussed has served to highlight the existence of the logarithmic and linear representations in both child and adult samples. Remember, however, that the Overlapping Waves Theory states that different strategies can be utilized to complete a task. Thus, in number line estimation studies, children, say on the 0-1000 task, have been reported to progress from a logarithmic to linear representation, based on the strategy allowed for use, contingent on the amount of numerical information available for integration into estimates (Ashcraft & Moore, 2011). Also, these strategies may compete with each other, thus allowing for a linear response by a child on the 0-100 task, while concurrently allowing the expression of a logarithmic response on the 0-1000 task. To delve deeper into this progression, another conceptual component must be explained, since the developmental literature in number line estimation reviewed here indicates that the ability to view explicit parameters alone does not necessarily elicit linear responding in children. Specifically, I refer to a strategy that has been termed a midpoint strategy (Petito, 1990).

The midpoint strategy was first proposed by Petito (1990) to qualitatively describe counting strategies that were observed during a number line task. Specifically, Petito reported a strategy in which children would routinely fixate on the middle of the line before making their estimates. This was thought to indicate the integration of conceptual number knowledge of the numerical half of the continuum tested. Petito also reported that the use of this strategy increased with age of the participants tested. Schneider et al. (2008) reported a quantitative measurement of this strategy through the use of eye tracking methods. The results from this investigation showed the same increasing reliance on a midpoint strategy; the frequency of the fixations at the midpoint increased with age.

Recently, this strategy has been reported to develop alongside the development of strong linear responding. Ashcraft and Moore (2011) found that 100% of children in 3rd grade exhibited linear responding on the 0-100 number line estimation task. Interestingly, they also report the concurrent development of a midpoint strategy, as shown through the mean error profiles of the grade's estimates. Here, the errors associated with the estimates along the 0-100 continuum were significantly lower at the midpoint of the continuum than at the surrounding quartile regions of the line. These data seem to suggest that a sophisticated strategy, calculating the midpoint based on the information provided by the origin and endpoint of the number line, might be a critical cognitive component necessary for adult-like, linear responding, since they have been found to develop at or around the same time in development. Indeed, the mean error plots of the 2nd grade children who showed linear appearing estimates did not show evidence of the midpoint strategy.

As stated previously, the Overlapping Waves Theory can easily accommodate the developmental trends outlined here. The goal, then, is trying to extend this developmental trend and theory to an adult sample to try to verify the cognitive components thought to elicit linear responding in children. The next section will attempt to provide an account that will extend this framework to adult performance in estimation.

The linear representation seems to be a product of mathematics education, with taught principles of equidistant numbers, while the logarithmic representation seems to be a product of our "number sense", or intuitive understanding of number. Children have been shown to develop a linear representation after sufficient education is acquired, which helps map the objective magnitudes of number in a linear fashion. This is evidenced from Dehaene's work, developmental work in number line estimation, and Siegler's work with teaching the rules of a linear board game, aiding in children's performance on number line tasks (Siegler & Ramani, 2009). Note that children have demonstrated the ability to express either representation, depending on the availability of salient numerical information. The linear representation is found in participants who have acquired enough educational experience to become, at a minimum, familiar with the parameters of the task. This is shown in developmental data where children in the second grade hold both logarithmic and linear representations, and the expression of these representations is dependent upon the context of the parameter. With increasing age and educational experience, all of the children will adopt the linear representation to complete the same number line tasks, with increasing cognitive sophistication as demonstrated in the use of a midpoint strategy.

The logarithmic representation is found in situations in which the parameter is "fuzzy" (where exact numbers are not present, as opposed to a parameter that is not perceivable). This has been shown in numerosity estimation where there is no mention of actual number, although arrays of dots represent the parameter, (Dehaene et al., 2008), and dot enumeration of magnitudes ranging from 0 to 100 (Izard & Dehaene, 2008). In both of these studies, the parameter is not explicitly stated, as in the typical number line estimation task.

I make the distinction of "fuzzy" versus unknown parameters because of the robust finding of scalar variability (Dehaene, 2001; Dehaene & Marques, 2002). This idea encompasses data that are characterized as having a somewhat linear pattern of responses, but as the estimate moves further from 0 (the only known parameter), the variance about the points of estimation increases monotonically. Izard and Dehaene (2008) explain scalar variability as the product of the idiosyncratic understanding of number when no calibration is present (calibration refers to exposure to numerical properties of the task, leading to familiarization to the parameter tested).

The logarithmic representation is a product of our "number sense". By number sense, I mean those situations in which no number labels or salient parameters are present. The logarithmic representation is expressed in situations in which the relation of quantities or numbers has precedence and the linear representation is expressed in situations that involve direct observation and application of taught principles of number. However, it is important to note that it is not enough to claim that the logarithmic representation is an approximate system. This conception of number sense is plausible, given logarithmic representations in adults have only been shown in conditions that provide inexact parameters, and other evidence of logarithmic thinking has been shown to be the only representation for children of certain ages who have not received formal training, cultures with no formal education, and lower animals. The Mundurucú, regardless of age or level of education within the tribe, show the logarithmic pattern, because they have no verbal system to differentiate between individual quantities above a certain magnitude. For example, Dehaene et al. (2008) provided the example that 9 is verbalized as "a handful and four on the side", and beyond this range of numerosity, the magnitudes are expressed as being "many". It is clear that the Mundurucú must represent quantities by category or approximation. These populations have no means by which to estimate the parameters of a task, and I argue this is the reason why logarithmic representations are expressed.

Again, the present proposal aims to actively manipulate the expression of the linear and logarithmic representations held by adults through the framework of the Overlapping Waves Theory (Chen & Siegler, 2003; Siegler, 1996). This framework provides an account describing the progression from an analog, primitive logarithmic representation to an advanced, sophisticated, linear representation of number. More specifically, the theory states that children possess working knowledge of many different strategies to complete any one task at any one time throughout development. The amount of experience and information the child has about the task at hand can change the strength of certain strategy use, and with more practice, more efficient and advantageous approaches to a problem gain precedence in strategy selection.

This framework makes no claim of purging less efficient strategies from memory, but instead posits the assumption that less efficient and less frequently used strategies possess a weaker signal of activation compared to the more efficient alternative. With these assumptions in mind, and given the results of the literature examined here, it seems plausible that the amount of information one possesses about the characteristics of the parameter is crucial in the estimation process. By exploring this possibility in adults, who already have the means by which to produce linear responding, the current work is designed to better understand the cognitive components crucial to accurate estimation in the development of estimation strategies, and how these strategies lead to the expression of separate representations.

CHAPTER 2

PILOT EXPERIMENT

To investigate parameter knowledge in dot enumeration, an experimental design was constructed such that a yes/no signal detection paradigm was combined with an array estimation task. In short, two groups viewed collections of dots ranging from 30 to 1000 dots, accompanied by numerical labels. The groups were the Parameter group (which received numerical information relating to the endpoint of the distribution tested) and the No Parameter group (which did not receive any numerical information about the task). The accompanying labels displayed two types of numerical information. For the control trials (30 to 60 dots), the labels corresponded to a value that was 5% above and 5% below the actual number of dots in each collection. For the test trials (100 to 1000 dots), two numerical labels were also created. The set of linear labels corresponded to the actual number of dots displayed in each collection. The logarithmic labels corresponded to a logarithmic transform of the actual number of dots in each collection. The procedure of this design was as follows.

On each trial the participant indicated, with a “yes” or “no” judgment, if the numerical label accurately described the number of dots in each collection. In the event that the participants would respond “no”, they were then asked to provide a numerical estimate that they believed to be an accurate description of the number of dots presented on that trial. The control stimuli (collections of 30 to 60 dots) served as a between subjects control that was expected to receive equivalent levels of “yes” responding between groups. This control was included to provide a standard for between groups comparisons. Pilot data collected in this manner suggested that the two methods, when combined, may not have been able to fully address the hypotheses outlined above.

This conclusion was supported upon examining the participants' responses to the control stimuli. Analyses conducted to examine the group differences of "yes" responding to the control stimuli indicated that the Parameter and No Parameter groups were approaching a significant difference of "yes" to "no" responses (t (47) = 1.9, p = .06), showing that parameter knowledge did have a considerable amount of influence on the rate of "yes" responding to this range. Because the proposed design resulted in control stimuli responding that was nearly significantly different, the procedure was split into two separate experiments: pure magnitude estimation and pure signal detection. Although it was speculated that these two tasks may rely on different processes to address numerical understanding and representation, these processes were still thought to rely on a common numerical mechanism for the judgment of magnitude. With this in mind, the following outlines the design of the current experiments.

Experiment 1 (pure magnitude estimation) examined the differences in estimation ability between the Parameter and No Parameter groups to investigate the influence of endpoint knowledge in dot enumeration. These data indicated that the endpoint alone was not sufficient to elicit linear responding in adults, although considerable differences were observed between the accuracy and reaction time data provided by the two groups. This experiment also provided important information as to how the parameter should have been defined. This led to the notion that the midpoint of the distribution should also be saliently recognized to elicit linear responding.

Experiment 2 (pure signal detection) also examined the influence of endpoint information, but in terms of its contribution to the ability to state preference between the linear and logarithmic sets of labels applied to the collections of dots, reflecting the influence of representation utilized. These data, like Experiment 1, also indicated that the endpoint information alone is not sufficient to elicit linear responses. Instead they supported the interpretations drawn from the pure magnitude production task: that the midpoint is a crucial component of linear responding.

The key conclusion drawn from the data obtained in Experiments 1 and 2 is that the parameter should not just include the origin and endpoint information as previously argued. Instead, it is argued that origin and endpoint knowledge alone only set the stage for logarithmic responding. Also, from the results obtained in the present investigation, along with convergent evidence from the literature, support the conclusion that salient understanding or calculation of the midpoint is crucial for the formation and utilization of a linear representation in adults and children alike.

CHAPTER 3

EXPERIMENT 1

METHOD

Participants

Participants were drawn from the undergraduate subject pool at the University of Nevada Las Vegas. A total of 44 college adults (Mean age = 19.63; 17 males, 27 females) participated in this experiment. Participants were issued research credit required for partial fulfillment of their Psychology 101 course requirements.

Stimuli

The stimuli created for this task were 62 dot arrays of magnitudes ranging from 100 to 1000 dots. Each stimulus consisted of a lone white circle 415 pixels in diameter, presented upon a black background. Within this circle were dot arrays that corresponded to the 62 magnitudes within the 100 through 1000 range. The dot arrays were created using computer software, and the exact location of each dot was determined by random clicking within the white field, with each click placing a black dot on the image. The program was able to count the number of clicks applied to each image, and the experimenter would monitor the total number of clicks, stopping when the desired amount of dots were included into the image. Thus, the arrangement of the dots was static for each stimulus, and did not change across participants. *Figure 2* presents a sample stimulus from this experiment. The exact magnitudes presented in the test distribution were: 100, 122, 147, 150, 163, 179, 190, 209, 213, 225, 246, 268, 284, 307, 310, 326, 342, 350, 366, 389, 408, 412, 423, 457, 472, 486, 490, 510, 525, 541, 564, 575, 581, 590, 607, 614, 627, 650, 660, 681, 690, 710, 725, 738, 744, 754, 778, 790, 802, 818, 839, 845, 861, 876, 890, 901, 910, 920, 938, 951, 960, and 980. Additionally, because Experiment 2 tests the same range of 100 to 1000 dots in the context of a signal detection task, an additional set of control trials were needed for that experiment to compute *d′*. To maximize the comparability of the two experiments, the same control trials needed for Experiment 2 were included here as well: 62 trials of dots with each of the magnitudes 30 through 60, presented twice each.

Procedure

Each participant was randomly assigned to one of two between group conditions. The Parameter group (N = 21) was provided with explicit information regarding the range of magnitudes to be estimated in the experiment, while the No Parameter group (N = 23) was unaware of this range. Both groups were provided with instructions indicating that they would be viewing a series of figures that consisted of differing numbers of dots. Both groups were told to view the collection and, as quickly and accurately as possible, to provide a numerical estimate that was thought to correspond to each collection. Next, participants were given a sample trial. The Parameter group's sample trial displayed an array that consisted of 1000 dots, and were told to view the array and to note that this array represented 1000 dots, the maximum amount of dots possible in this experiment. The No Parameter group was simply shown a blank field, with the word "dots" surrounded in brackets in the center. Both groups were also told to note the duration of this sample slide, as it would correspond to the presentation time of the stimuli during the experiment (2 seconds).

After the sample stimulus presentation, the Parameter group was reminded of the range of magnitudes (0-1000) and was also reminded to estimate as quickly and accurately as possible. The No Parameter group was given the same instruction, with the exclusion of the range of magnitudes tested. Two practice trials corresponding to 30 and 342 dots were provided prior to the experimental trials. The stimuli were presented in a mixed-block design experiment, which was administered using the E-prime 2.0 software (Schneider, Eschman, & Zuccolotto, 2002), and participants spoke their estimates into a microphone. When the verbal response was registered by the microphone, the latency of response was recorded. A total of 124 observations were recorded per participant. Before analyzing the data from Experiment 1, outlier analyses were conducted on the reaction times associated with the estimates (Dixon, 1953). A total of 394 trials, or 9.6% of the entire set of observations were removed.

CHAPTER 4

RESULTS

The results for Experiment 1 will focus solely on the 62 test magnitudes; however, to support the use of the control trials in Experiment 2, analyses were conducted on the 31 control trials presented to participants in Experiment 1. This 2 (group: Parameter and No Parameter) X 31 (control stimuli magnitudes) repeated measures ANOVA demonstrated that the estimates provided by the two groups did not significantly differ within this range (F (1, 42) = 2.02, *n.s.*). The implication of this result will be addressed in Experiment 2.

The results section for Experiment 1 is organized in the following manner. First, I present the overall performance of the two groups in terms of the R^2 fits of the linear and logarithmic models of curve estimation regression. Then, I will present performance in the form of absolute error and RT analyses to help clarify the importance of a crucial aspect of linear estimation—the midpoint knowledge.

Estimates

To analyze the estimation data, each group's mean response to all test stimuli were entered into curve estimation regression to determine the best fit of the data, yielding R^2 values corresponding to the linear and logarithmic model fits. *Figure 3* presents the mean estimates of the two groups.

As can be seen from *Figure 3*, the No Parameter group grossly underestimated the amounts of dots in each array, however a linear increase in response was observed with an increase in the numerosity of the stimuli. This pattern of response was expected. For example, many dot array enumeration studies have demonstrated similar patterns of response and have been attributed to scalar variability (Izard & Dehaene, 2008; Krueger, 1982; 1984). The curve estimation regression analysis indicated that the mean response of this group was not very well fit to either model, however the R^2 values were equal for both the linear and logarithmic fits (R^2 Linear = .76; R^2 Logarithmic = .76). In contrast, the Parameter group provided estimates that better approximated the actual value of each stimulus. The curve estimation regressions calculated for the Parameter group's mean performance showed much stronger fits to the data, with a higher R^2 value for the logarithmic model than the linear model (R^2 Linear = .93; R^2 Logarithmic = .97). Note, that since each participant only saw each stimulus once, individual participant's R^2 fits were not very informative due to high variability. This is in contrast to Izard and Dehaene (2008), where participants were provided with 600 trials, allowing for individual analyses. Not surprisingly, these two patterns of estimates differed significantly in a 2 (group) X 62 (test magnitudes) Repeated Measures ANOVA: F (1, 42) = 9.43, $p < .01$, $\eta^2 = .18$).

The estimation data show that without calibration, participants are unable to accurately estimate the total number of dots in each array. The R^2 fits for the No Parameter group were not well fit by either the linear or logarithmic models. With calibration, the Parameter group was much more accurate in their estimates, and also responded in a manner that was best fit by the logarithmic model. These data replicate Izard and Dehaene (2008) in their investigation of base level calibration to magnitudes within the range of 0-100. To further investigate the benefits of calibration in this task, I will now present the error data associated with these estimates. This type of analysis has been performed in number line estimation (Ashcraft & Moore, 2011), and revealed many interesting patterns leading to deeper insight into the cognitive processes utilized during estimation. As such, these same procedures were conducted for the current investigation.

Error Analyses

Figure 4 presents the mean error associated with each of the 62 dot arrays presented in this experiment, split by group. First, notice the overall estimation differences between the groups. Although similar in the first half of the continuum, the two groups differ substantially beyond the midpoint (the split appears to begin at 525 dots). To analyze these differences in error between groups, a 2 (group) X 62 (magnitudes) repeated measures ANOVA was computed. This analysis yielded a main effect of group (F (1, 42) = 26.62, $p < .01$, $\eta_p^2 = .39$) a main effect of magnitude (F (61, 2562) = 9.21, $p < .01$, $\eta_p^2 = .18$), and a significant group by magnitude interaction (F (61, 2562) = 5.94, $p < .01$, $\eta_p^2 = .12$). These analyses describe that the Parameter group estimated with significantly less error than the No Parameter group (mean Parameter group error = 201.39; mean No Parameter group error = 359.09).

It is important to note that the divergence in absolute error occurs around the midpoint, as depicted in *Figure 4*. From this figure it is apparent that the last point at which the two groups show similar levels of error is at the magnitude 525. This is the first piece of evidence from this study that highlights the importance of the knowledge of the midpoint. The No Parameter group, with no knowledge of the numerical properties of the task, continues to estimate with increasing error as the magnitudes exceed the midpoint values. The Parameter group, with knowledge of the endpoint value, shows a similar increase of error to the midpoint, then remains relatively unchanged to the endpoint. Thus, showing that endpoint knowledge did not appreciably change the way the Parameter group evaluated the midpoint region of the continuum. I return to the implications of this result throughout this work.

To minimize any potential concerns with inflated degrees of freedom, and also to highlight the main regions of the continuum of interest here, the number continuum was divided into thirds to further investigate the error patterns seen in *Figure 4*. These thirds were created by calculating the mean responses for all estimates corresponding to the magnitudes within the three semi-equal regions of the continuum. This resulted in the creation of origin, midpoint, and endpoint thirds. The origin third consisted of 20 values ranging from 100 to 389 dots, the midpoint third consisted of 22 magnitudes ranging from 408 to 710 dots, and the endpoint third consisted of 20 magnitudes ranging from 725 to 980 dots. This procedure was modeled after Ashcraft and Moore (2011), in which estimates within the quartile regions of the 0-100 and 0-1000 number line task were averaged to provide a less cluttered approach to the data. The reason for isolating entire thirds across the continuum in the present study was due to the nature of the task; i.e., that participants had to make estimates along a continuum of values without visual reference points, leading to higher variability in the data than found in the conventional number line task.

With this procedure in mind, a 2 (group) X 3 (thirds) repeated measures ANOVA was computed to investigate the differences in error along the continuum. This analysis paralleled the one considering all 62 magnitudes, revealing a significant main effect of group (F (1, 42) = 27.26, $p < .01$, $\eta_p^2 = .39$) a significant main effect of thirds (F (2, 84) =

74.45, $p < .01$, $\eta_p^2 = .64$) and a significant group X thirds interaction (F (2, 84) = 44.54, $p < .01$, $\eta_p^2 = .52$). *Figure 5* plots this interaction.

These analyses show virtually no difference in estimation within the origin third of the continuum. This is interesting given the different amounts of knowledge the groups were able to integrate into their estimates. Although the No Parameter group underestimated while the Parameter group overestimated, the departure from perfect estimation was almost identical for the two groups even up to the midpoint. Within the midpoint third of the continuum, however, post-hoc comparisons indicated that there was a major divergence in error patterns. Although both groups demonstrated significant increases of error, the No Parameter group estimated with significantly more error than the Parameter group within this third. Note that the data from all magnitudes within the midpoint third were reflected in the analysis, thus incorporating both the convergent and divergent responses within this third observed in *Figure 4*. In the endpoint third of the continuum, the No Parameter group showed a significant increase in error, while the Parameter group showed a non-significant decrease in error within this range.

These trends show the particular benefit of the endpoint knowledge while making numerosity estimates of dot arrays ranging from 0-1000, resulting in less estimation error. Interestingly, however, this benefit is only observed once participants are estimating magnitudes at or beyond the midpoint of the continuum. This raises important questions regarding the role of midpoint knowledge in an estimation task because origin and endpoint knowledge did not change midpoint estimation. The next portion of this work outlines the results obtained from the reaction times associated with these estimates to further address these midpoint questions.

Reaction Time

In line with the error analyses, the RT data were evaluated through a 2 (group) X 62 (test magnitudes) repeated measures ANOVA. This analysis revealed a significant main effect of group (F (1, 42) = 5.83, $p < .05$, $\eta_p^2 = .12$) a non-significant main effect of magnitude across the continuum (F (61, 2562) = 1.21, n.s.) and a significant magnitude X group interaction (F (61, 2562) = 1.81, $p < .01$, $\eta_p^2 = .04$). *Figure 6* plots this interaction. As can be seen in the figure, the RTs are highly variable. This is thought to be due to the difficulty of the task (creating judgments for large arrays without any visual aid, such as a number line). Despite this variability an interesting pattern emerged. Specifically, it was observed that there are two distinct areas of separation between group's RTs (the origin and endpoint thirds of the continuum) and one area of convergence (the midpoint third). Because of this observation, parallel thirds procedures and analyses were performed for the RT data as were performed for the error data. The next section of the results outlines the findings of these procedures.

To analyze the groups' RTs among the thirds, a 2 (group) X 3 (thirds) repeated measures ANOVA was computed. These calculations revealed a significant main effect of group (F (1, 42) = 5.94, $p < .05$, $\eta_p^2 = .12$) a non-significant main effect of thirds (F (2, 84) = 1.34, n.s.) and a significant thirds X group interaction, (F (2, 84) = 3.85, $p < .05$, $\eta_p^2 = .08$). *Figure 7* presents the interaction from this analysis. The figure shows a large separation in overall RT, with slight reaction time differences among the thirds for each group. Repeated Measures ANOVAs were computed separately for the two groups to investigate the differences in RT along the thirds within each group. This analysis was significant for the Parameter group (F (2, 40) = 5.79, $p < .05$, $\eta_p^2 = .22$), but was not significant for the No Parameter group (*F* (2, 40) = .837, *n.s.*). Bonferroni post-hoc comparisons indicated that the Parameter group demonstrated a significant decrease from the midpoint to the endpoint third, indicating quicker judgments in the endpoint third. The comparison from the origin to midpoint third was not significant.

These analyses provide an interesting perspective with regard to adult enumeration. First, it is not surprising that the No Parameter group would take longer overall to produce estimates of numerosity for the various magnitudes, but it is interesting to note that this is even true within the origin third, where error around the estimates were virtually identical. So, despite the similar level of accuracy, it is important to note that the No Parameter group takes about a half of a second longer, on average, to provide their responses within this range.

A second point of interest lies in the group performance along the thirds. The No Parameter group did not show a significant latency difference along the thirds, indicating that the lack of numerical knowledge resulted in similar levels of processing across the entire continuum. The Parameter group, though, was aided in the ability to produce estimates, with overall faster RTs. Although endpoint knowledge globally affected estimates along the entire continuum, as Izard and Dehaene (2008) showed with origin information, processing speed was only truly speeded for the endpoint third, although some speeding was observed for the origin estimates as well.

The final and most compelling trend to be discussed for the RT data is, actually, the non-significant difference between the groups' RTs within the midpoint third. The effect is especially clear in *Figure 6*, RTs across all 62 magnitudes. As noted earlier, two areas of between subject comparisons showed a large degree of separation of parameter and no parameter RTs. These two gaps are present in the origin and endpoint thirds of the continuum. Interestingly, however, the RTs of the two groups did not differ significantly within the midpoint third of the continuum ($t(42) = -1.71$, *n.s.*). This trend is thought to demonstrate the mutual difficulty of the two groups to produce estimates within this range of number without having explicit exposure to the midpoint prior to testing.

CHAPTER 5

EXPERIMENT 1 DISCUSSION

Experiment 1 investigated how explicit numerical knowledge about the endpoint of a continuum ranging from 0-1000 would influence numerosity judgments. Again, two groups (Parameter and No Parameter) were asked to provide estimates to collections of dots within this range, and this manipulation of endpoint knowledge resulted in interesting differences between the groups. First, the No Parameter's mean estimates were not well fit to either the linear or logarithmic models, as predicted by curve estimation regression, while the Parameter group's mean estimates were best fit by the logarithmic model. The lack of fit within the No Parameter group is believed to reflect idiosyncratic responses of the participants, (Izard & Dehaene, 2008). Because they had no information relevant to the task, the No Parameter group was unable to judge the stimuli in comparison to an objective standard. This is in contrast to the Parameter group's estimates which were best fit by the logarithmic model. By calibrating their mental number line in reference to the endpoint, estimates were much more accurate, and tended to follow a logarithmic pattern. This logarithmic set of responses was also observed in Izard and Dehaene (2008), and is also found in developmental number line estimation tasks, reportedly because of the limited amount of number knowledge children can integrate into estimates (Ashcraft & Moore, 2011; Booth & Siegler, 2006; Siegler & Opfer, 2003).

The absolute errors and RTs associated with the estimates of the two groups revealed interesting patterns as well. Regardless of the level of explicit number knowledge related to this task, both groups showed equivalent increases in error to magnitudes up to the midpoint. At this point, the error functions diverged, with a progressively increasing error function beyond the midpoint in the No Parameter group, and an error function that slightly decreased beyond the midpoint in the Parameter group.

The RT data show significantly faster responses when explicit number knowledge was integrated into numerical estimates, as compared to performance when this information was not present. Although this was true for overall performance, analyses on the thirds along the continuum revealed that these differences were not universal along the entire continuum. Also, despite the differences in explicit number knowledge about the continuum, the Parameter and No Parameter groups responded with similar RTs within the midpoint third.

The estimation data from Experiment 1 seem to suggest that both groups respond similarly up to the midpoint third of the continuum. This is a compelling observation given the original predictions of this project: that "parameter" knowledge would result in linear responding in contrast to the logarithmic pattern that was actually observed in the Parameter of this study. From the onset of this work, the "parameter" was defined as explicit information regarding the origin and endpoint of the continuum being judged. This logic was drawn from the number line estimation literature (Ashcraft & Moore, 2011), in which error and latency patterns indicate strategies of response progressing from an origin-up process (1st grade), to a more advanced strategy utilizing both explicitly labeled origin and endpoint (2nd grade), to the adult-like strategy of utilizing both endpoints to form a representation of the midpoint, which is also utilized as a reference point (3rd grade).

The factor overlooked in transferring children's number line estimation data to adult's performance in the current dot enumeration study was the static image of the number line, or a physical representation of the continuum. The logic followed that, because adults already demonstrate proficient knowledge of the range of magnitudes in this task (0-1000) then basic knowledge of the parameter would yield linear responding. However, as just stated, the physical representation of the magnitude was not presented in this task. So, although adults demonstrate knowledge of the midpoint of the range of number (500), this knowledge cannot be transferred to a physical representation of 500 dots, since they have not been previously exposed to the image for calibration.

Results from Experiment 1 seem to suggest that salient exposure to the midpoint representation of 500 dots might be the remaining cognitive component necessary to elicit linear responding. In terms of absolute error, divergence within the midpoint between the two groups' functions indicate that the working knowledge used by both groups is equivalent up to the midpoint (where endpoint knowledge is then used in the Parameter group), and the RT data demonstrate equivalent processing speeds to provide estimates within the midpoint third, compared to different processing speed within the origin and endpoint thirds. Experiment 2, then, was conducted to see if a signal detection method would yield corroborating evidence in the direction of Experiment 1's trends.

CHAPTER 6

EXPERIMENT 2

To gain a different perspective of the implication of acquiring endpoint knowledge for numerical judgments, a yes/no signal detection task was conducted in addition to the estimation task of Experiment 1. Experiment 2 investigates the implicit influence of one representation over another while stating preferences to the same dot arrays presented in Experiment 1. The stimuli for Experiment 2 were the same dot arrays as viewed in Experiment 1, with the addition of a numerical label that either represented the linear representation (or actual number of dots present), or the logarithmic representation (logarithmic transform of the linear amount of dots presented).

The logic for this task was that if participants are adopting or utilizing one representation over another, dependent on the amount of numerical information present, then the yes/no judgments provided should reflect the representation used during testing. Another possibility to be explored was if participants in the No Parameter group would show judgments resulting in a pattern showing little or no preference for one label type over another. This curiosity was raised in Experiment 1, where the No Parameter group responded in a way that is supportive of scalar variability, or idiosyncratic responding (Izard & Dehaene, 2008). Thus, this task is investigating the ability of participants to discriminate between the two sets of labels provided, in favor of the type of representation utilized during test.

This task was designed to support the conclusions drawn from the data in Experiment 1. When estimating numbers of dots in each collection without explicit numerical knowledge about the task, the No Parameter group responded in a way that was not consistent with either the linear or logarithmic representation. Conversely, when explicit number knowledge was provided, the Parameter group responded in a way that was strongly consistent with the logarithmic representation. If representational use is an implicit tendency to view numbers in patterned ways depending on the amount of numerical information present to form numerical judgments, then this task should parallel the results obtained in Experiment 1.

CHAPTER 7

METHOD

Participants

A total of 61 participants (Mean age = 21.92; 22 Males, 39 Females) were drawn from the University of Nevada Las Vegas subject pool. Each participant was compensated with research credit toward the completion of the Psychology 101 course requirements.

Stimuli

In Experiment 2, the same set of 93 magnitudes was presented as in Experiment 1. In Experiment 2, however both distributions of number (control and test magnitudes) were of interest for analysis. The control distribution consisted of collections ranging from 30 to 60 in magnitude, while the test distribution collections ranged from 100 to 1000 dots. One modification was made to the stimuli for the signal detection task: the inclusion of a numerical label. This label consisted of a white rectangle placed above the field of dots explained for Experiment 1. Different types of numerical information were inserted into the labels, which are described in the following section.

For the test distribution of magnitudes, two numerical labels were applied to each stimulus, one corresponding to the linear representation, and one corresponding to the logarithmic label. The linear label provided a number that was the exact match of dots in each array, whereas the logarithmic label was the log-transformed value of the exact number of dots, determined through the function $y = (1/.0069)\text{Ln}(x)$. This particular function was adopted from Siegler and Opfer (2003), explaining the ideal logarithmic responder derived from child performance in 0-1000 number line estimation. Thus for a collection of 100 dots, the linear label read "100", while the logarithmic label read "667". *Table 1* presents the linear and logarithmic labels for each test magnitude presented in this task.

The control stimuli were also accompanied by two numerical labels. One label displayed a number that was 5% above the actual number of dots displayed, while the other label displayed a number that was 5% below the actual number of dots in the collection. The purpose of the control stimuli in this task was to provide a set of stimuli that would rely on the same process as the test distribution, but that would not be influenced by parameter knowledge. Because Experiment 1 revealed similar estimates, this conceptualization of a control distribution was substantiated. Thus, this distribution was created to provide a measurable standard by which performance could be compared both within and between groups, to determine the varying levels of agreement between the two test distribution labels.

Procedure

As in Experiment 1, each participant was randomly assigned to one of two groups: the Parameter or No Parameter group. Again, the Parameter group was given explicit information regarding the endpoint of the continuum through the same instructions included in Experiment 1. The only addition to the endpoint slide was a linear label of "1000" above the collection of dots of the same magnitude. Likewise, the No Parameter group was simply shown a slide highlighting the format of the stimuli, with the word "dots" within the collection field, and "number" within the label field. Both were surrounded by brackets. The participants were also told to note the duration of the instruction slide, as it would represent the duration of each stimulus in the experiment (2 seconds). Instructions emphasized speed and accuracy equally.

The stimuli were presented within a yes/no signal detection paradigm. Participants were instructed to judge the appropriateness of each numerical label/dot collection pairing, and respond with a “yes” response if they felt that the label and collection pair matched, and conversely to respond “no” if they did not match. Instructions specified that the participant press the leftmost button of a stimulus response (S/R) box if they agreed with the label/collection pairing, and to press the rightmost button if they disagreed with the pairing. Each stimulus was present on the screen for 2 seconds, at which point the screen turned black until the participant provided a response. After a decision was provided, a ready prompt appeared for 2 seconds, indicating the coming of another stimulus. Again, the stimuli were presented in a mixed-block design, presented via E-prime software (Schneider, Eschman, & Zuccolotto, 2002). This design yielded a total of 186 observations per participant.

As was performed in Experiment 1, outlier detection took place before analyzing the data, through Dixon’s (1953) test. This procedure was only performed for the RTs, and resulted in 214 total latencies removed, or 2.1 % of the entire sample. Responses associated with the reaction times were not removed.

CHAPTER 8

RESULTS

Because this experiment was designed as a yes/no signal detection paradigm, d' is the statistic of interest. d' measures the discriminability between the theoretical distributions of representation.

Discriminability was measured through the function:

$$d' = z\,(\text{hits}) - z\,(\text{false alarms})$$

The size of d' is interpreted as the theoretical distance between the two peaks of signal distributions in decision space. The larger the d' value translates to larger distances between the representations of number in this task.

An exclusion criterion of "yes" responding, across all stimuli, that was less than 20% or greater than 80% was set prior to data collection. Performance described by these criteria was assumed to reflect negatively on the participant's motivation at time of test, and also assumes that the participant was not making credible judgments throughout the experiment. This criterion resulted in the exclusion of five participants from analyses, leaving a total of 56 participants in the analyses (29 in the Parameter group, and 27 in the No Parameter group).

Discrimination

Before presenting the d' data, I first present results that establish the control stimuli as a viable standard for the comparison of the test distributions. To validate between-group similarity in response to control stimuli, an independent samples *t*-test was computed on the ratio of "yes" to "no" responding. This analysis was non-significant, $t(54) = -.93$, $p =$

.36. In the Parameter group, the mean percent of "yes" response was 68% overall, and in the No Parameter group, the mean percent of "yes" response was 73%. In other words, regardless of the amount of numerical information made available to the participants, an equivalent percentage of "yes" responding was present, thus establishing the control distribution as an acceptable between subjects control for later tests of discrimination. Again, this conclusion is also supported by the estimation data observed in Experiment 1.

Overall *d′*

To test for evidence of representational influence across all test stimuli in Experiment 2, *d′*s were computed to measure the level of discriminability between the distributions. The *d′*s computed speak to the ability of the participants to indicate their overall preference for the linear and logarithmic labels that accompany the collections of dots. Thus, overall preference for logarithmic labels would indicate influence of the logarithmic representation, and linear representation influence would be indicated if participants demonstrated overall preference to the linear labels.

To investigate the overall preference for label types, three *d′* measures were computed for each group. These *d′*s corresponded to the discrimination between the linear and control distributions (LinCon *d′*), the discrimination between the logarithmic and control distributions (LogCon *d′*), and the discrimination between the linear and logarithmic distributions (LinLog *d′*). The LinLog *d′*s are of most interest in this study, as they speak to the ability of the groups to indicate preference for one type of label over the other. If a participant is using the linear representation while making the yes/no judgments, then the *d′* value obtained would result in a positive integer. This is because the proportion of linear “yes” responses used in the d' calculation was entered into the formula as “Hits” and the proportion of logarithmic label “yes” responses was entered as “False Alarms”. Thus, a negative d' for a LinLog d' would indicate a larger overall preference for logarithmic labels above the overall preference for linear labels.

The LinCon and LogCon d's were calculated to enable between group comparisons of discrimination. Without these measures, the LinLog d' would only provide the basic information of label preference, not the extent by which the two groups prefer the linear or logarithmic labels compared to a control. Thus, the LinCon and LogCon d's allow for the comparison of label preference, indicating if the preference for the logarithmic label, for example, was stronger in the Parameter or No Parameter group.

To compare the overall discrimination of the two groups, a one-way ANOVA was calculated on the overall LinLog d's obtained from the data. This analyses revealed a significant main effect of group, F (1, 54) = 5.56, $p < .05$. Surprisingly, this analysis showed that the No Parameter group, devoid of numerical information about the task, showed higher levels of discrimination than the Parameter group (mean No Parameter group $d' = .32$; Parameter group $d' = -.15$). This result was unexpected, as it seems to imply that viewing the parameter of the continuum resulted in less discrimination than not seeing any numerical information at all, and that discrimination observed in either group was not very strong. The data from Experiment 1 certainly speak against this claim, showing that parameter knowledge increased accuracy in estimation of these magnitudes, and also led to a logarithmic trend in estimates. To help clarify this possibility, refer to *Figure 8a* and 8*b*, Parameter and No Parameter “yes” responses along the entire continuum.

These two figures display the ratio of "yes" responses to the linear and logarithmic labels along the continuum of magnitudes tested, separately by group. The trends of the lines speak to the level of agreement of each label presented for each magnitude. Also note that the reference line placed at .5 indicates responding at chance level, thus, deviation below the reference line describes a low level of "yes" responding for that specific label type, while deviation above the reference line describes a high level "yes" responding for that specific label type. Inspection of the two graphs reveal that the two groups do respond to the labels differently along the entire continuum however, within the first portion of the graphs, both groups seem to show better than chance discrimination, favoring the linear labels.

More specifically, the trends of both groups describe high level of agreement for the linear labels accompanying magnitudes of the continuum close to the origin (100), and concurrently demonstrate low levels of agreement to the logarithmic labels accompanying these same magnitudes. It is important to note that these two lines are not reflections of each other. On the contrary, they are independent of each other—reflecting the ratio of "yes" responding to the linear and logarithmic labels accompanying each collection of dots. With this in mind, it is easy to see that the discrimination of the distributions (linear vs. logarithmic) is substantial for both groups in the portion of magnitudes closest to the origin, as there is strong agreement for one label type (linear) and weak agreement for the other label type (logarithmic).

Next, view *Figure* 8*b*, ratio of "yes" responding along the continuum within the No Parameter group. Despite strong discrimination between the distributions near the origin, the ratio of "yes" responding for both labels converges around magnitude 423, and also, "yes" responding is centered on the reference, or chance line. Two things are occurring after this convergence. First, the fact that the two lines converge indicates that there is low discrimination between the labels applied to each collection of dots. In other words, participants in the No Parameter group were unable to differentiate between the linear and logarithmic labels applied to the magnitudes. Take the magnitude, 541 as an example. The convergence of these two lines indicate that participants were unable overall to respond "yes" to the linear label of "541" more often than the logarithmic label of "912". Second, the lines converge around the chance line, indicating that, on top of lack of discrimination, the No Parameter group was unable to respond to either type of label better than chance.

The trends observed in the No Parameter group are in contrast to the trends observed in *Figure 8a*, ratio of "yes" responding along the continuum within the Parameter group. As mentioned earlier, the Parameter group also responded with strong discrimination between the two labels types applied to the magnitudes closest to the origin, in favor of the linear labels. Beyond this trend lies an interesting pattern. The d' values reported earlier suggest that the Parameter group was less able than the No Parameter group to discriminate between the distributions applied to the magnitudes, but closer inspection of *Figure 8a* reveals that, in fact, the Parameter group actually does discriminate between these label types. Surprisingly, preference for the linear labels decreased, while preference for the logarithmic labels increased beyond the magnitudes closest to the origin. This is consistent with Experiment 1 in which endpoint knowledge elicited a logarithmic response. It seems plausible that this reversal was the reason for the lack of discrimination indicated with the overall d' measure, that is, the differential preference for distinct portions of the continuum led to an averaging effect, or seemingly low discrimination.

After discrimination between the label types applied to the magnitudes closest to the origin, favoring the linear distribution, the Parameter group demonstrated a reversal in label type preference. Thus, instead of responding with little discrimination along the continuum as the overall *d′* suggested, *Figure 8a* reveals strong discrimination along the continuum, with a reversal in representation preference, dependent on the location of the stimuli within the continuum. This realization, then, led to the same splitting procedure reported in Experiment 1. Splitting the performance of the groups allows the investigation of this reversal by focusing the analyses on the same semi-equal regions created in Experiment 1, which also provides the means by which to compare the regions among the two experiments. The exact splits reported earlier were also calculated in Experiment 2 and the next section outlines the results revealed by this procedure, first separately by group then in terms of the between group differences.

Parameter Group Discrimination by Thirds

I begin by examining the LinLog *d′*s within the Parameter group. Again, this comparison speaks to the within group preference of the two test distributions. *Figure 9* presents the *d′* results along the three portions of the continuum (origin, midpoint and endpoint thirds), for the Parameter and No Parameter groups. Here, note that the reference line (placed at the 0 mark on the y-axis) refers to no discrimination between the labels applied to the magnitudes.

The overall trend of the figure shows strong discrimination with a preference for the linear labels over the logarithmic labels in the origin third (*d′* = .71). The opposite relationship is true in the midpoint third (*d′* = -.67), showing strong discrimination between the distributions with a preference for the logarithmic labels. Performance in the endpoint third shows a slight decrease in discrimination between the two test distributions, while still holding a preference for logarithmic labels (*d′* = -.52). A repeated measures ANOVA computed on the thirds indicated that this trend was significant (F (2, 56) = 27.76, $p < .01$, $\eta^2 = .49$). Bonferroni post-hoc comparisons indicated that the LinLog *d′* difference from the origin to the midpoint was significant, while the difference between the midpoint and endpoint was not, showing that the level of discriminability and preference did not change from the midpoint to the final third.

These analyses demonstrate a significant cross-over interaction in the Parameter group's performance from the origin to the midpoint thirds. Specifically, "yes" responding to the linear labels was significantly higher than the logarithmic labels in the origin third, while the opposite was true in the midpoint third. To verify these trends, a 2 (LinCon *d′* and LogCon *d′*) x 2 (origin and midpoint thirds) repeated measures ANOVA was computed to see if the interaction was still present when comparing the test distributions to the control within these two thirds. This analysis confirmed the cross-over, demonstrating a non-significant main effect of thirds (F (1, 28) = .02, *n.s.*), a significant main effect of *d′* (F (1, 28) = 5.88, $p < .05$, $\eta_p^2 = .17$) and a significant *d′* X thirds interaction (F (1, 28) = 36.50, $p < .01$, $\eta_p^2 = .57$). These results not only indicate that agreement was higher for the linear labels within the origin third, while agreement was higher for the logarithmic labels in the midpoint third, but that the Parameter group has completely switched which type of label was preferred, depending on the location of the continuum being judged.

As could be expected following the estimation trends of Experiment 1, the influence of the endpoint knowledge was not enough to elicit judgments favoring linear labels in this task. What is interesting, however, is that the Parameter group switched from a linear preference of number in the origin third, to a logarithmic preference of number within the midpoint and endpoint thirds of the continuum. This provides additional evidence that the salience of the midpoint is critical for linear responding in estimation tasks. Next to follow is the No Parameter group's performance on this task to investigate how participants performed when no numerical information was available.

No Parameter Group Discrimination by Thirds

The data collected from the No Parameter group underwent the same splitting procedure as described for the Parameter group. The following section details the trends in d' described by thirds of the stimulus continuum. Specifically, the aspect of interest was to see if the splitting procedure would be more informative than the overall d' statistic provided earlier.

As can be seen in *Figure 9*, the same degree of discrimination between the linear and logarithmic labels in the Parameter group was also found in the origin third for the No Parameter group, with significantly more "yes" responses for the linear labels than for the logarithmic labels. However, instead of demonstrating the reversal in distribution preference from the origin to midpoint thirds, the No Parameter group demonstrated low, and perhaps even no discrimination within the second and final thirds of the continuum.

Despite this lack of discrimination within the midpoint and endpoint thirds of the continuum, the overall trend is significant (F (2, 52) = 34.34, p < .01, η_p^2 = .57). Post-hoc analyses revealed a significant difference in responding from the origin to the midpoint, but not from the midpoint to endpoint thirds.

The figure displays that after the strong discrimination that favored the linear labels in the origin third of the continuum (d' = 1.18), participants in the No Parameter group displayed little to no discrimination in the midpoint third (d' = .02), with responding virtually falling on the chance line. This is followed by a slight increase in discrimination within the endpoint third (d' = -.24). However, this increase is still not significantly different from chance, shown through a one-sample t-test on the LinLog d's for this third (t(26) = -1.49, *n.s.*).

Discrimination Differences between Groups

To investigate the between groups differences among the LinLog d's along the thirds, a 2 (group) x 3 (LinLog d's) repeated measures ANOVA was computed. This analysis revealed a significant main effect of group (F (1, 54) = 5.04, p < .05, η_p^2 = .09), a significant main effect of LinLog d' by thirds (F (2, 108) = 59.65, p < .01, η_p^2 = .53), and a non-significant group x LinLog d' interaction (F (2, 108) = 1.12, *n.s.*). Post-hoc analyses investigating the differences of d' by each third indicated that there were no differences between group within the origin and endpoint thirds, but a significant difference in the d's between the two groups within the midpoint third.

The between groups d' results show that the overall trends of discrimination between the Parameter and No Parameter groups were not similar, as the original d' data implied, and that overall discriminability was higher in the Parameter group than in the No Parameter group. In terms of individual comparisons along the thirds, the two groups show equivalent discrimination between the linear and logarithmic labels when the number of dots viewed ranged from 100 to 389 (origin third), and also when the number of dots ranged from 725 to 980 dots (endpoint third). It may seem counterintuitive that the groups would not differ in the endpoint third since the Parameter group was given a visual representation of the endpoint, however a possibility for the similarity in endpoint responding is that the two label types were closer within the endpoint third than anywhere else along the continuum. This would lead both groups to be more likely to respond "yes" to any stimulus within this range, because each label still approximates the end of the continuum. This can be seen in the non-significant upward trend of "yes" responding in both groups in *Figures 8a* and *8b*. Regardless, the influence of the endpoint knowledge can be observed in the data, as the Parameter group's discrimination was better than chance beyond the origin third, whereas the No Parameter group's discrimination was not better than chance responding as demonstrated by one-sample *t*-tests (Parameter group endpoint d': $t\ (28) = -\ 4.32$, $p < .01$; No Parameter group endpoint d': $t\ (26) = -\ 1.49$, *n.s.*).

In addition to evidence supporting the influential role of endpoint knowledge, the discrimination data provide additional support for the notion that midpoint knowledge is especially important for the formation of linear strategy use. Specifically, between groups comparisons of d' show a significant difference within the midpoint third, but not in the origin or endpoint thirds. Not only did the two groups differ in terms of discriminability, but the Parameter group provided responses supporting the logarithmic labels more than the linear labels, while the No Parameter group demonstrated chance-level responding with no detectable bias after the origin third of the continuum. The fact that the Parameter group shows a complete cross-over from linear to logarithmic responding from the origin to the midpoint third is intriguing. This suggests that endpoint knowledge provides participants with enough cognitive influence to guide detection in a way that is superior to guessing, but not enough cognitive influence to make the linear judgment throughout the entire continuum. Experiment 1 also suggested this conclusion. To view the influence of endpoint knowledge in processing time, the RTs associated with the judgments of each stimulus was analyzed. The next section describes the results of this investigation.

Reaction Time

The trends of the reaction time data associated with the yes/no judgments demonstrated the following patterns: contingent on the type of label seen (linear or logarithmic), the Parameter and No Parameter groups showed differential RT patterns, which were also dependent on the response chosen (yes or no). These patterns were also differentially expressed, based on the location of the stimulus being judged along the entire continuum (thirds). *Figure 10* displays these trends.

Within the Parameter group, facilitation in response time was seen along the thirds for “yes” responses, while hesitation in response time was observed for the “no” responses. Facilitation refers to a steady decline in RT as the magnitude to be judged approaches the endpoint. Hesitation refers to a plateau, or at least a trend that does not display a steady decline in RT as the magnitudes increase. Also, the Parameter group shows faster overall reaction time for logarithmic labels than linear labels. In contrast, the No Parameter group demonstrated facilitation in responding for “no” responses, while concurrently demonstrating hesitation for “yes” responses. The No Parameter group also showed faster overall RTs to the logarithmic labels, similar to the Parameter group.

These trends describe the patterns found to describe the 4-way interaction observed in the RT data. This 2 (distribution: lin/log) x 2 (response: yes/no) x 2 (group: parameter/no parameter) x 3 (thirds: origin/midpoint/endpoint) interaction was significant, $F\ (2, 108) = 3.94$, $p < .05$, $\eta_p^2 = .07$. The F statistics associated with the lower-order effects of this interaction are presented in *Table 2*. Next, I will describe these trends in more detail and discuss the possible implications drawn from these relationships.

Facilitation in Reaction Time

The first effect to be explained here is the tendency for both groups to show levels of facilitation in responding along the thirds. Facilitation refers to larger reaction times in the origin third, with intermediate RTs in the midpoint third of the continuum, and fastest RTs within the endpoint third of the continuum. The Parameter group demonstrates this trend while making “yes” responses, and the No Parameter group responds in this fashion while making “no” responses. Accompanying each facilitation pattern is a plateau-like trend, such that the Parameter group provides a flatter RT function for “no” responses, while the No Parameter group provides these RT trends for “yes” responses.

These results speak to the preparedness, or perspective, with which the two groups respond in the task. Because the Parameter group received numerical information about the task, it follows that this information would prepare participants to confirm the relationship between the labels and collections. In other words, the Parameter group is able to integrate task specific numerical information into the pre-existing information gained throughout years of experience with numerosity. In contrast, the participants who did not receive any numerical information about the task would be best able to disconfirm any label/collection pairing, relying solely on past numerical information to base their judgments. It is thought that this disconfirmation process relies on idiosyncratic numerical mappings, as Izard and Dehaene (2008) have suggested for participants who have not been calibrated to the continuum tested. Along with these differential patterns is the observation that the overall RT means of hesitant responses are significantly longer than the overall RT means for the facilitation responses.

Group

Apparent in *Figure 10* is the overall faster reaction time of the Parameter group to respond to all stimuli than the No Parameter group. The explanation provided here is that knowledge of the parameter speeds processing because the reference point has been integrated into judgments. This was apparent in the *d′* results as well. The Parameter group was able to use this endpoint knowledge to respond with more confidence to both label types.

The No Parameter group took longer to respond because they had less numerical information to work with, judging preference from previously seen stimuli, as opposed to basing preference on a framework utilizing explicit number properties of the task. They could only construct the parameter through perception, as they had not seen the endpoint, and could only infer the size of magnitudes based on previous trials seen in the experiment. This subjective criterion for judgment is more taxing than responding based on a visual standard for comparison. The slower response is also believed to be the byproduct of the lack of discrimination demonstrated by the No Parameter group.

Distribution

The main effect of distribution is apparent from the figure such that both "yes" and "no" responses are faster for the logarithmic labels than linear labels, for both groups. This is thought to reflect the "intuitive" appeal of these labels, as it is thought to correspond to our primitive, analog understanding of number (Dehaene, 1997; Geary et al., 2007). In other words, it seems possible that viewing a logarithmic label actually taps into the default understanding of number, so to speak. In the Parameter group, overall responding favored the logarithmic labels. Here, it seems possible that quicker response could relate to the confidence with which the group responded to these labels, as it corresponds to the representation used during test. This effect is particularly interesting in the No Parameter group, since both estimation and detection data revealed an overall inability to respond in a way that supported either representation. Thus, for the No Parameter group to respond faster to logarithmic labels than the linear labels, when both behavioral and explicit measures show poor performance, it then follows that the RT trend is reflecting the implicit understanding of number within this group. Although the participants do not show preference for the logarithmic representation in either experiment, they do show faster overall responding to this label type.

CHAPTER 9

EXPERIMENT 2 DISCUSSION

Experiment 2 was designed to investigate the implicit utilization of a representation while making yes/no judgments to two different types of labels applied to the same collections estimated in Experiment 1. Again, the critical assumption of this task was that if participants adopt or utilize one representation over another, which is dependent on the amount of numerical information present, then the yes/no judgments provided should favor the representation used during testing. Similar to the Izard and Dehaene (2008) study, this experiment investigated the influence of semantic number knowledge on dot enumeration.

The purpose of Experiment 2 was to determine if discriminability of stimuli labeled with numbers derived from linear and logarithmic functions would support the results obtained in the estimation experiment. Specifically, it was hypothesized that the No Parameter group would show little discrimination between the two label types, while the Parameter group would prefer the logarithmic labels accompanying the arrays. Thus, this task enabled the investigation of the influence of varying levels of number knowledge while verifying the numerical amount of items within large number arrays.

The results of this task show that semantic knowledge can, in fact, alter the precision with which participants can judge complex arrays of dots. Without any numerical information about the task, participants were largely incapable of discriminating between the sets of linear and logarithmic labels (discrimination was only seen for about 20 of the 62 stimuli). Participants provided with numerical information relevant to the task, however, adequately discriminated between these two functions along the entire continuum. Interestingly endpoint information alone led to overwhelmingly logarithmic label agreement after the origin third, reflecting the limited cognitive influence endpoint information lends to yes/no judgments. Perhaps with explicit information about the midpoint of the continuum, participants would prefer linear labels throughout the entire continuum, completing the replication of the developmental progression of cognitive influence, as seen in child estimation. This possibility remains to be tested.

CHAPTER 10

GENERAL DISCUSSION

Two experiments were conducted to investigate the influence of number knowledge in the enumeration of dot arrays within the range of 0-1000 dots. Two experiments investigated this influence. Experiment 1 addressed the influence of endpoint knowledge in a magnitude estimation task, where participants were asked to provide numerical estimates of arrays of dots. Experiment 2 also addressed this influence in a yes/no signal detection task, where participants were asked to judge the appropriateness of a numerical label placed above each array.

As found in previous dot enumeration studies, Experiment 1 showed the improvement in estimation due to minimal amounts of calibration to the continuum tested. Specifically, estimates provided when no calibration was presented were grossly inaccurate, with estimates of the arrays that overwhelmingly underestimated the objective value of the collections tested (Izard & Dehaene, 2008; Krueger, 1982; 1984). Also in concordance with the literature, estimates became much more accurate with calibration to the parameter (Izard & Dehaene, 2008). The RT data from Experiment 1 also demonstrated the benefit of calibration. Without numerical information about the task, the No Parameter group demonstrated a non-significant RT trend along the continuum, while the Parameter group, who did receive numerical information about the endpoint, demonstrated a significant trend. In terms of overall RT, calibration led to a significant difference between the groups, with the Parameter group responding significantly faster than the No Parameter group. These data showed that endpoint knowledge not only facilitated estimation ability overall, but also processing speed of the stimuli, especially within the origin and endpoint thirds of the continuum.

Experiment 1 also highlighted the important role midpoint knowledge may have in forming a linear representation of number. The error profiles associated with estimates to the stimuli provided by the two groups were indistinguishable for values up to the midpoint, after which the profiles diverged, indicating that evaluation of the collections leading up to the midpoint was similar in both groups. The RT data also suggested this apparent similarity in midpoint assessment. Between group comparisons along the thirds indicated significant mean RT differences in the origin and endpoint thirds, but not in the midpoint third, reflecting similar cognitive effort required by both groups to evaluate the numerosity of this region of the continuum when explicit instruction did not highlight the midpoint, or numerical half of the continuum.

Experiment 2 also examined the importance of endpoint knowledge in evaluating this range of number. With no numerical knowledge available to base judgments of the stimuli, participants demonstrated a level of discrimination across the entire continuum that was little better than chance. When given endpoint knowledge, however, participants were able to discriminate between the labels throughout the entire continuum. The interesting component of the Parameter group's performance was that discrimination in favor of the linear labels was observed in the origin third, followed by a significant cross-over in response from the origin to the midpoint, favoring logarithmic labels. This cross-over was sustained within the endpoint third as well.

The data from Experiments 1 and 2 demonstrate two compelling components of number understanding. First, this study clearly shows the cognitive influence observed when semantic knowledge of number is activated. Converging evidence was provided from two different methods to investigate this influence; estimation and detection results demonstrated the benefit of incorporating the endpoint knowledge into numerical judgments. These results replicated the effects shown by Izard and Dehaene (2008), in which participants were given a calibration stimulus and were told, (truthfully or deceptively) that the array contained 30 dots (30% of the total continuum). The current study lends support to the Izard and Dehaene study, showing a large change in estimation accuracy with the presentation of minimal amounts of calibration to the task. The current study, however, allows for a more detailed look into the development of a linear response, as it calibrates participants' judgments to a region thought to be important in the progression to linear responses in children: the endpoint.

The progression from a logarithmic to a linear representation in children has been the focus of many recent investigations and has also been shown to interact with later math achievement in school settings (Ashcraft & Moore, 2011; Booth & Siegler, 2006; Siegler & Opfer, 2003; Siegler & Ramani, 2008). In one such study, Ashcraft and Moore (2011) showed error functions relating to 1^{st} and 2^{nd} grade estimates on the 0-100 number line estimation task that are quite similar to the error functions produced by the adults in the present study. In their study, 1^{st} grade children demonstrated a gradually increasing error function, even beyond the midpoint. The 2^{nd} grade sample, however, showed an increase in error up to the midpoint, and then a steady decline in error as the hatch marks approached the endpoint. These trends were explained as relating to the estimation strategies utilized, based on the amount of numerical information understood by the child. In 1^{st} grade, the relevant strategy was determined to be a working forward process, producing estimates based on the distance of the hatch mark from the origin, with little semantic information influencing estimates. The 2nd grade strategy was explained as a process in which the nearest endpoint from the hatch mark was the basis for judging magnitude, thus utilizing both the origin and endpoint values to make estimates. Both grades do show a decline in error as the hatch marks approached the endpoint, but this is thought to be the result of not being able to respond beyond the endpoint of the continuum, since the parameter was explicitly labeled on each trial. Importantly, 1st graders did not provide evidence of incorporating the numerical properties of the endpoint into estimates.

The progression of more sophisticated strategies used by children in the above study reflects the principles outlined in the Overlapping Waves Theory. The strategies defined by Ashcraft and Moore (2011) gain additional sophistication upon the development of increasing levels of mathematics knowledge appreciated by the children in each grade. A similar progression of strategy use is believed to have been observed in the data provided by the adults in the current study. The No Parameter group, like the 1st grade children, demonstrated increasing error as the collections estimated approached the endpoint. Without knowing the exact endpoint, however, participants were solely reliant on their origin representation. Remember, both groups responded similarly to the control collections in both experiments, and this is the range of number thought to be the basis for all other judgments, thus forcing the No Parameter group to estimate in an origin-up fashion. The Parameter group participants, like the 2nd graders, are thought to be using the endpoint knowledge when the collection to be estimated was closer to the endpoint than the origin. This was indicated by the Parameter group's error profile, which slightly decreased once the collections exceeded the magnitudes of the midpoint. This provides evidence suggesting a strategy that utilizes both origin and endpoint information. Thus, as was found in the developmental literature, the acquisition of more specialized numerical information led to more sophisticated strategy use. Because the adults tested can be assumed to have the capability to respond with a high degree of sophistication (due to proficient number knowledge and estimation ability, Ashcraft & Moore, 2011), these data also highlight the principle of multiple strategy use, dependent on the amount of numerical information appreciated by the participant.

The second key point demonstrated by these experiments is the notion that the development of a linear representation seems to be heavily reliant on the salience with which the midpoint can be calculated and used to evaluate magnitude. Because the Parameter and No Parameter groups' performance has been shown to correspond to the developmental strategies utilized in 1st and 2nd grade, then it can be inferred that the strategies used by the adult participants in this task relied on the same cognitive components thought to influence the development of number sense in children. Assuming the truth in this logic then leads to the observation that neither of the adult groups provided linear responding, and it is argued here, that to achieve linearity, the participant must be able to recognize and integrate midpoint information into estimates. Specifically, I speak of a phenomenon observed in developmental number line estimation, the midpoint strategy (Ashcraft & Moore, 2011; Petito, 1990; Schneider et al., 2008). Interestingly, this has recently been shown to develop alongside of strong linear responding in the 3rd grade (Ashcraft & Moore, 2011).

In 3rd grade, 100% of the children in Ashcraft and Moore (2011) demonstrated linear responding and concurrently the emergence of a third, more sophisticated cognitive strategy: the midpoint strategy. This is characterized by the relative ease by which children can calculate and utilize midpoint information to further increase accuracy of estimates along the entire continuum of values tested. This strategy is indicated by a dip in estimation error along the midpoint values, producing an M-shaped error function. The M is thought to represent an "adult-like" understanding of number in the sense that the observer is fully aware of the numerical properties of the continuum tested. The emergence of the M-shaped error function indicates the use of both endpoints for the purpose of grounding estimates into a schema of number in which a third, more informative reference point can be created. Interestingly, although 100% of the 3rd graders demonstrated this type of strategy use through explicit behavioral measures (estimates), this strategy was not found to facilitate implicit processing of estimates (RT) until the child had become well practiced and familiar with the process, one year later in development. In the 4th grade, Ashcraft and Moore (2011) found this M function in estimates as well as reaction time, indicating the more proficient use of this strategy with increasing age and experience.

The results presented in this work strongly support the principles outlined in the Overlapping Waves Theory. More specifically, Experiments 1 and 2 attempted to capture the essence of the developmental progression of numerical understanding found in children, by recreating this progression in an adult sample. By doing so, these experiments have defined three crucial cognitive components necessary for linear estimation of magnitudes within a defined continuum of number: the appreciation of the origin, endpoint, and the utilization of calculated midpoint information.

As predicted by the Overlapping Waves Theory, the appreciation of these components led to a progression of estimation ability, which was the product of the type of strategy allowed for use, given the amount of numerical information made available to the participants. Like 1st grade children on the 0-100 number line task, the No Parameter group provided estimates that were characterized by an origin-up strategy, in which each stimulus was evaluated in reference to its distance from the origin of the continuum. As found in 2nd grade children, the Parameter group responded with a strategy characterized as integrating both endpoints of the continuum, with each stimulus being evaluated in reference to its perceived distance from the nearest endpoint (origin or endpoint). The implication of these findings is that the cognitive components manipulated in the current study actually caused the expression of patterns of response that mirror the developmental progression of children. Thus, these components can be manipulated in adults to better understand child understanding of number.

Concretely then, the present studies revealed intriguing parallels between adult performance in dot estimation and children's performance in number line estimation. Recent developmental work has outlined the cognitive strategies used by children to elicit the patterns found in the progression from the logarithmic to linear representation (Ashcraft & Moore, 2011), but previous studies have not attempted to break these strategies down into the crucial cognitive components necessary for their formation. This type of investigation is difficult in developmental populations because children have to develop the cognitive capacity necessary to accommodate these new strategies, which is largely dependent on factors outside of the control of experimenters. Thus, the current work demonstrated that these components can be investigated in adults, as adults have already achieved advanced numerical competency. In the current study, the No Parameter group gave estimates that resembled 1st graders' number line estimates, where each stimulus is evaluated in terms of its distance from the origin of the continuum. Using this strategy, errors simply increased across the continuum, and there was not a close correspondence between objective magnitude and the estimated magnitude. The Parameter group, however, was given the endpoint of the continuum, and made estimates that were not only more accurate, but were also much more rapid. The pattern of these estimates was logarithmic, and beyond the origin third of the continuum, the Parameter group showed a strong preference for the logarithmic labels. This performance clearly used information about both the origin and the endpoint of the continuum, as was true for 2nd graders, with each stimulus being evaluated with reference to the nearest endpoint of the continuum.

The third part of the developmental progression, a transition from logarithmic to linear responding, was accompanied by clear evidence that 3rd graders were relying on knowledge of the midpoint of the continuum. This third part of the progression was not found here with adults. Two possible reasons exist for this. One is that providing a display that showed the endpoint was insufficient for participants to form a mental representation of the midpoint of the continuum, and that displaying the midpoint might induce linear responding; a study that shows a midpoint display is currently in progress, to test this possibility. A second possibility is that the dot estimation task is so strongly reliant on perceptual processing that even adults will continue to base their judgments on largely perceptual information, and thus their numerical estimates will continue to rely heavily on the more primitive, logarithmic representation of magnitude. This perceptual processing is in contrast to the cognitive processing thought to be employed when making estimates with a midpoint strategy, where processing is reliant on mathematical relationships.

This work extended the Overlapping Waves Theory to create a progression of behavior and strategy use that closely approximates the cognitive trajectory found in developmental number line estimation. The next step in this research is to determine the exact cognitive component crucial for the development of the midpoint strategy. This strategy is associated with the full development of a linear representation, and thus, an adult-like understanding of number. By doing so, future research can attempt to examine these components and strategies in developmental samples to examine their individual contributions to later math achievement.

APPENDIX 1

TABLES AND FIGURES

Table 1
Linear and Logarithmic Labels of Magnitudes Tested.

Lin	Log	Lin	Log
100	667	575	920
122	696	581	922
147	723	590	924
150	726	607	928
163	738	614	930
179	751	627	933
190	760	650	938
209	774	660	940
213	776	681	945
225	785	690	947
246	797	710	951
268	810	725	954
284	819	738	957
307	829	744	958
310	831	754	960
326	838	778	964
342	845	790	966
350	848	802	969
366	855	818	972
389	864	839	975
408	871	845	976
412	872	861	979
423	876	876	981
457	887	890	984
472	892	901	986
486	896	910	987
490	897	920	989
510	903	938	991
525	907	951	993
541	912	960	995
564	918	980	998

Table 2

F-test Results of Experiment 2 Reaction Time.

Session (**A**): $F\ (1, 54) = 15.22$, $p < .01$, $\eta_p^2 = .22$

Distribution (**B**): $F\ (1,54) = 158.46$, $p < .01$, $\eta_p^2 = .75$

Response (**C**): $F\ (1, 54) = 4.77$, $p < .05$, $\eta_p^2 = .08$

Thirds (**D**): $F\ (2, 108) = 77.88$, $p < .01$, $\eta_p^2 = .59$

A x B: $F\ (1, 54) = 6.1$, $p < .05$, $\eta_p^2 = .102$

A x C: $F\ (1, 54) = 24.48$, $p < .01$, $\eta_p^2 = .31$

A x D: $F\ (2, 108) = 3.42$, $p < .05$, $\eta_p^2 = .06$

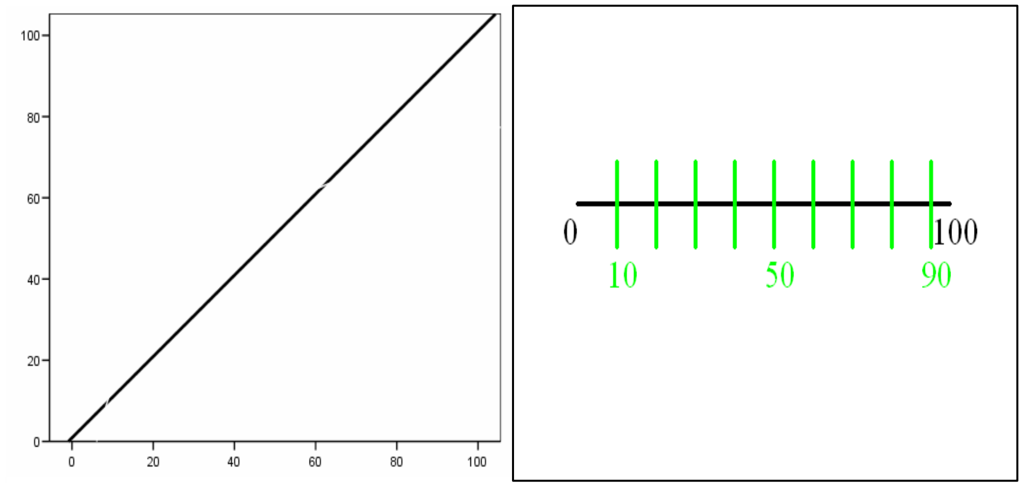
C x D: $F\ (2, 108) = 3.18$, $p < .05$, $\eta_p^2 = .06$

A x B x D: $F\ (2, 108) = 4.32$, $p < .05$, $\eta_p^2 = .074$

A x C x D: $F\ (2, 108) = 48.6$, $p < .01$, $\eta_p^2 = .47$

A x B x C x D: $F\ (2, 108) = 3.94$, $p < .05$, $\eta_p^2 = .068$

1a. Linear representation.



1b. Logarithmic representation.

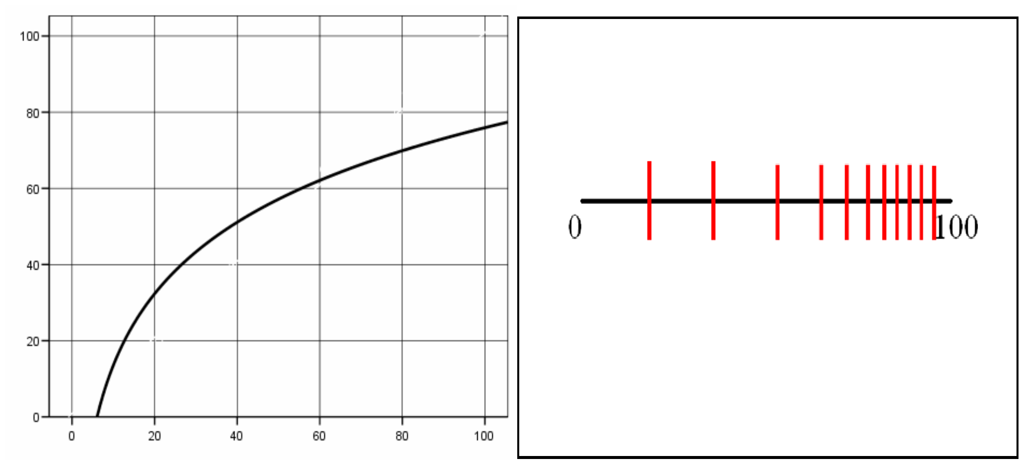


Fig. 1. Graphical representations of the linear(1a) and logarithmic representation(1b).

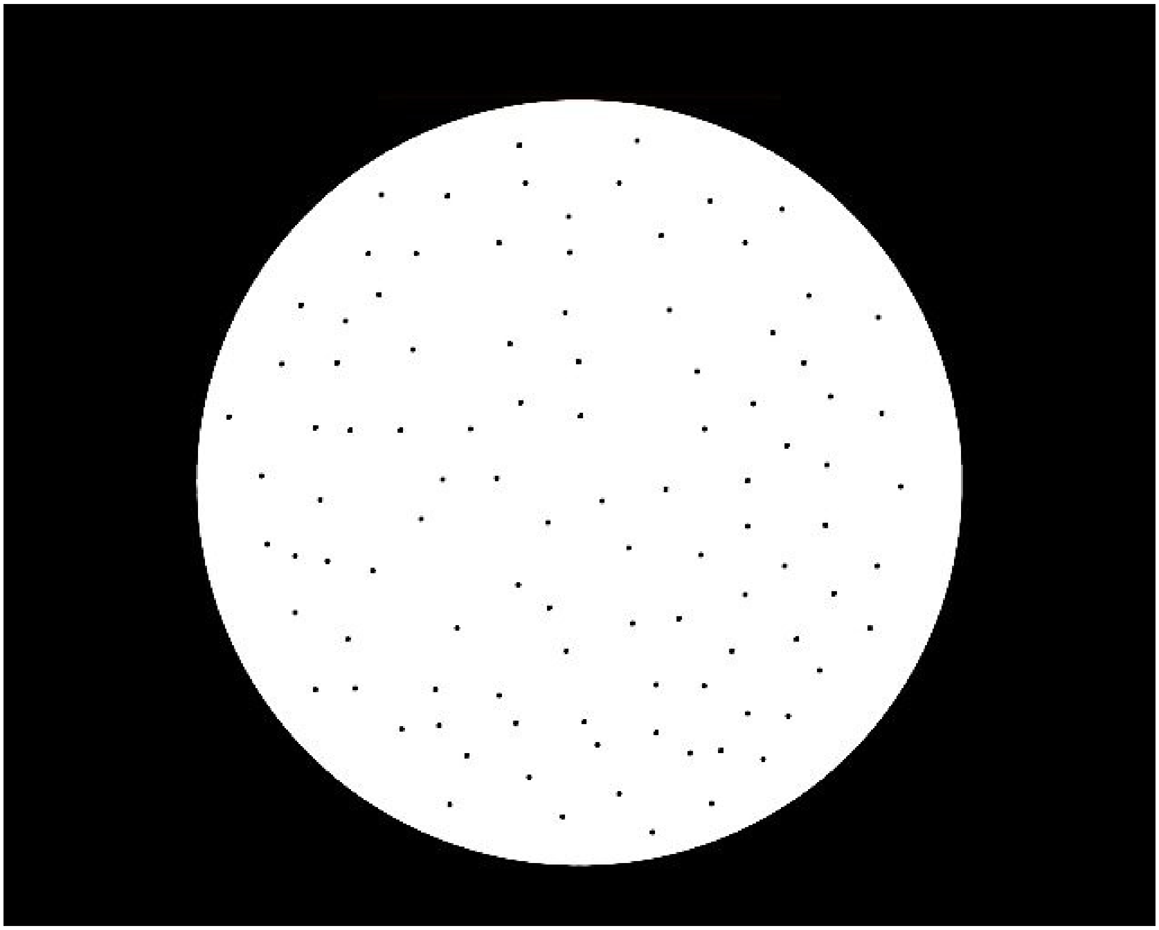


Fig. 2. Example target array (100 dots).

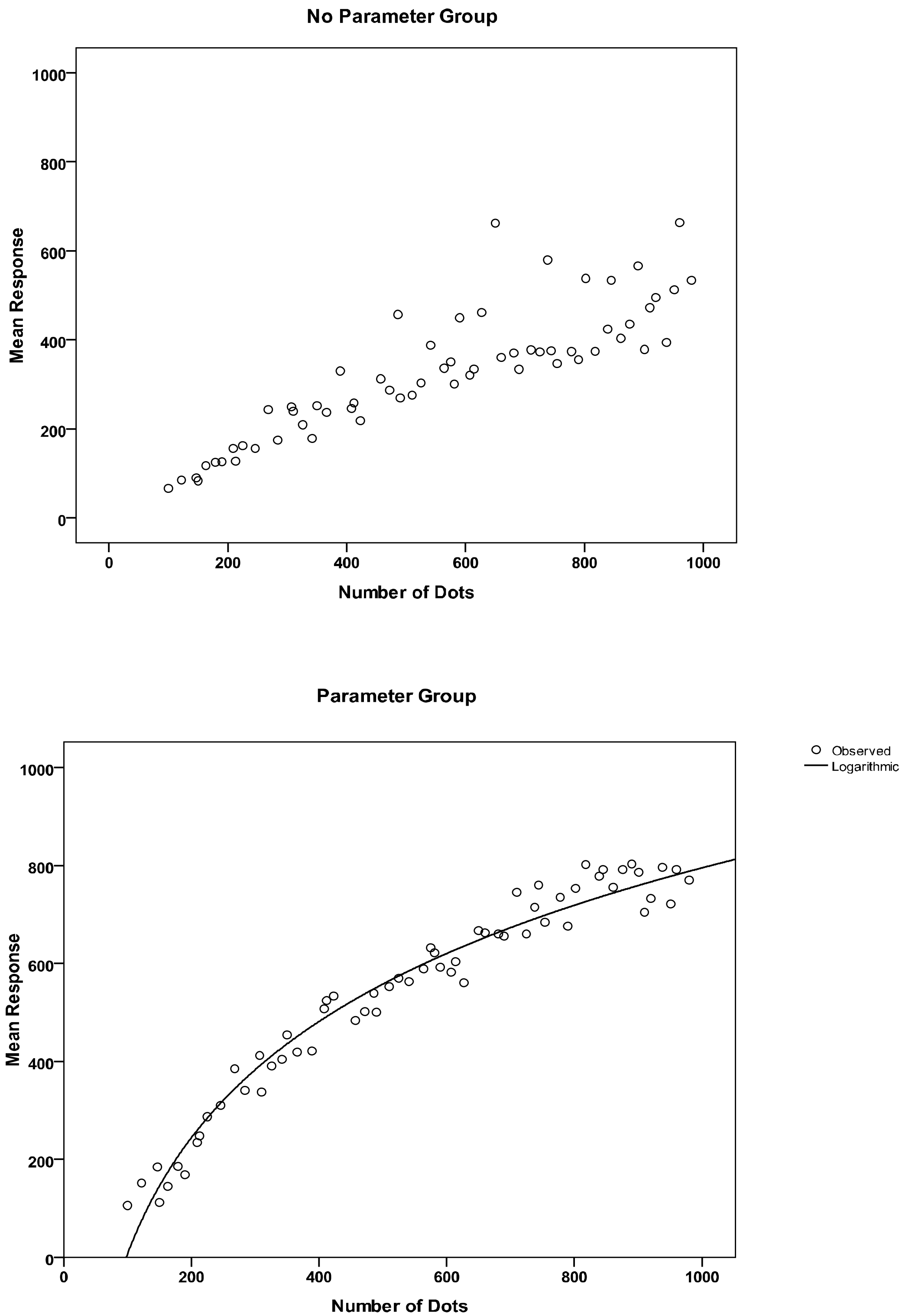


Fig. 3. Mean estimates for the No Parameter and Parameter groups.

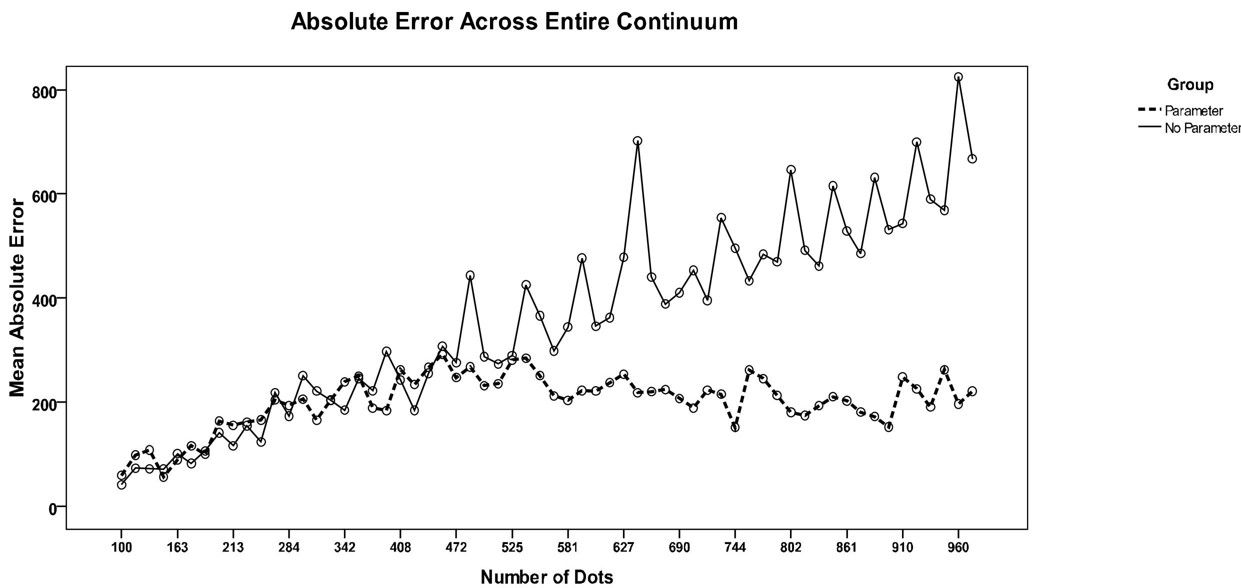


Fig. 4. Mean absolute error split by group.

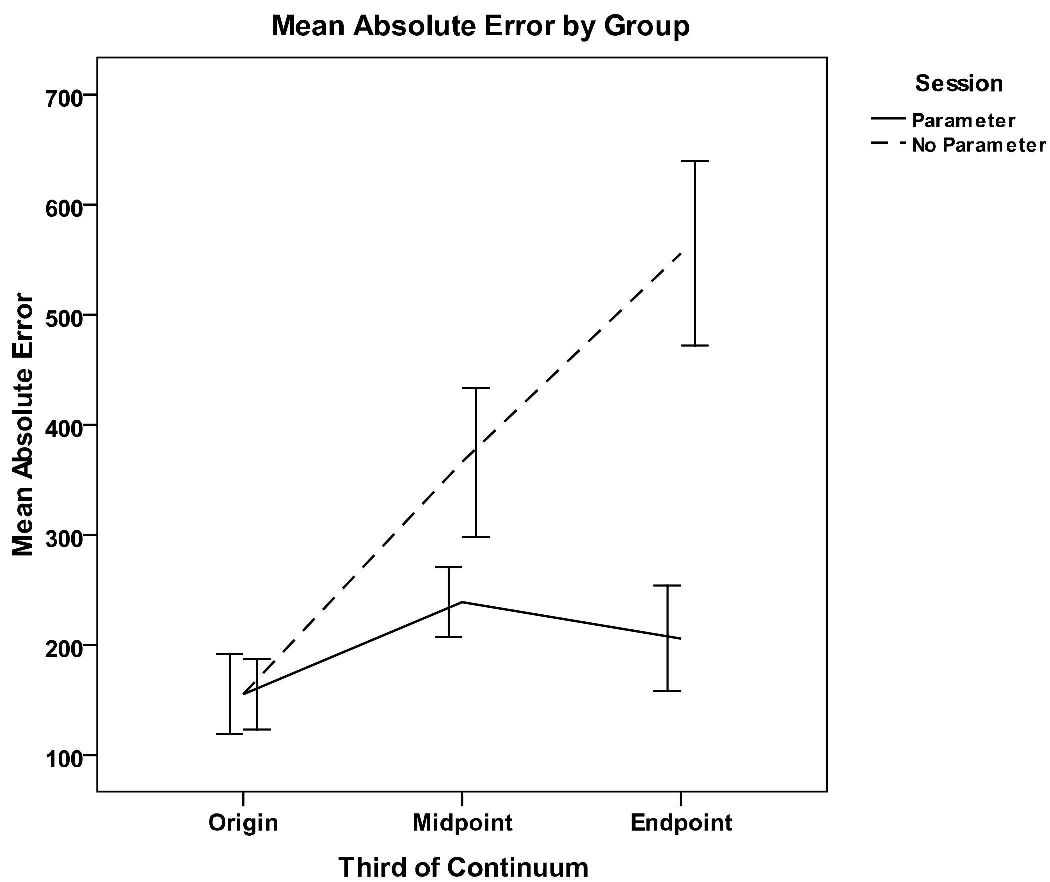


Fig. 5. Mean absolute error along the thirds split by group.

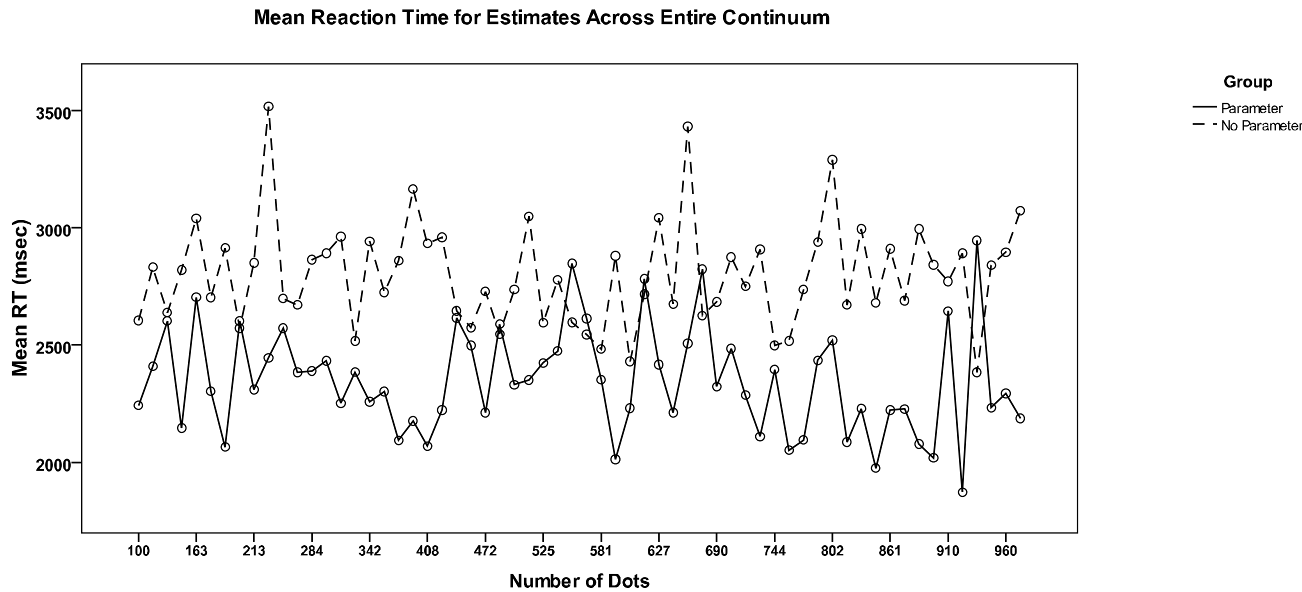


Fig. 6. Mean reaction time along the continuum split by group.

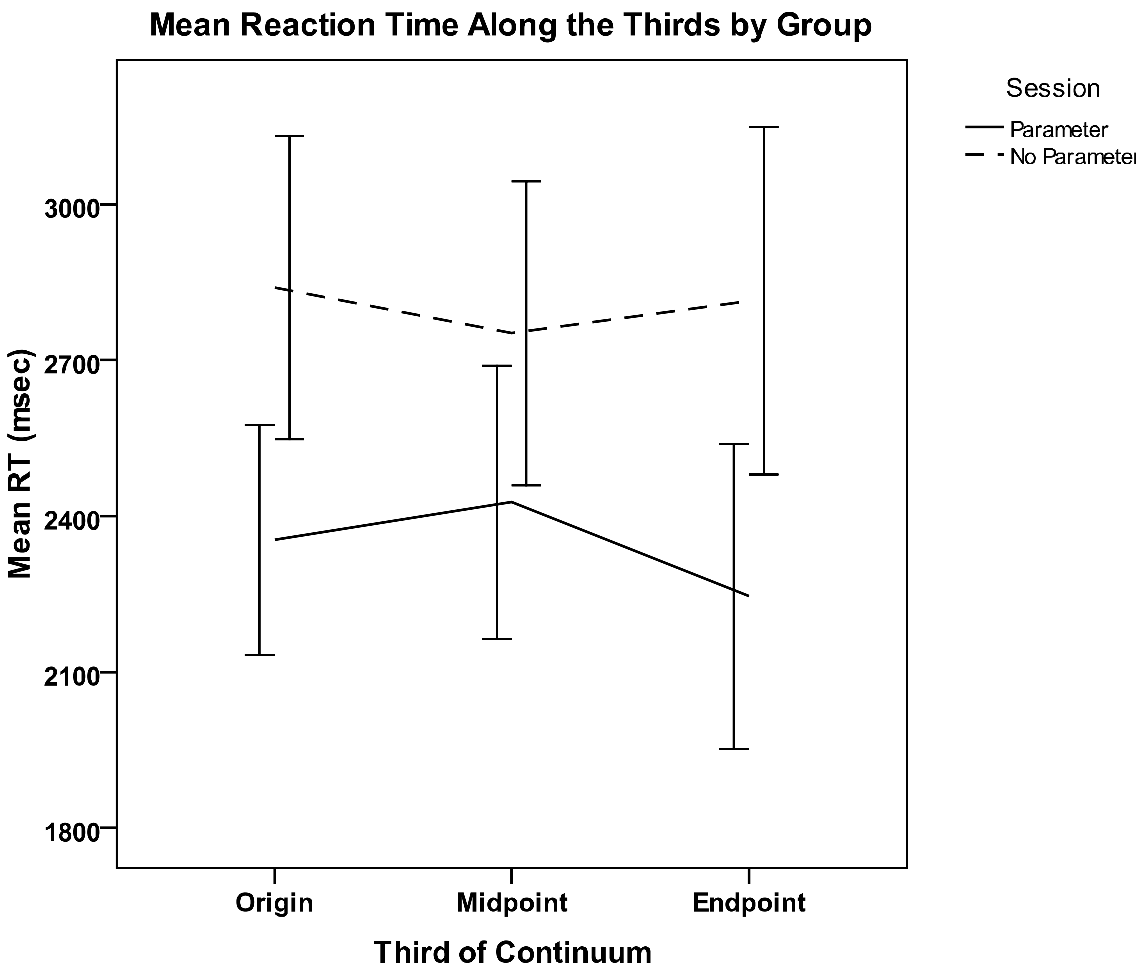


Fig. 7. Mean reaction time along the thirds split by group.

Fig. 8a.

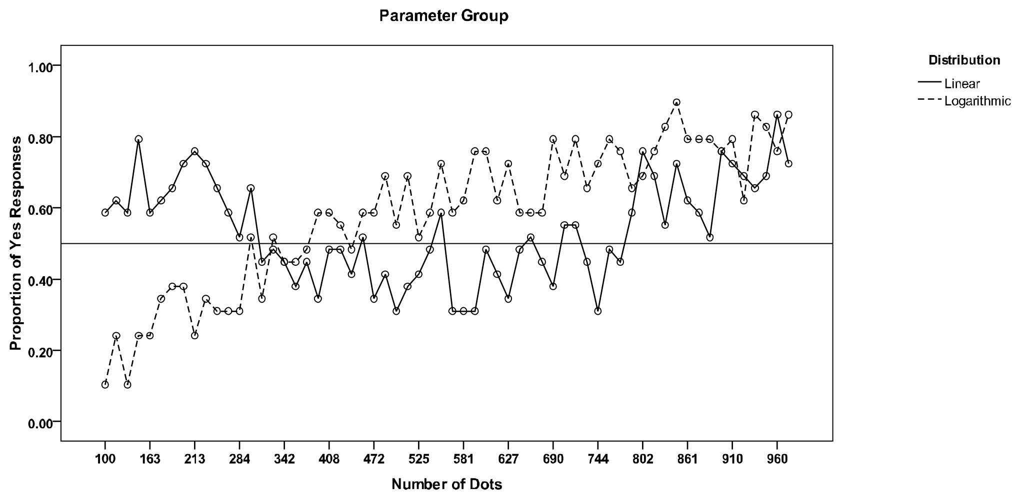


Fig. 8b.

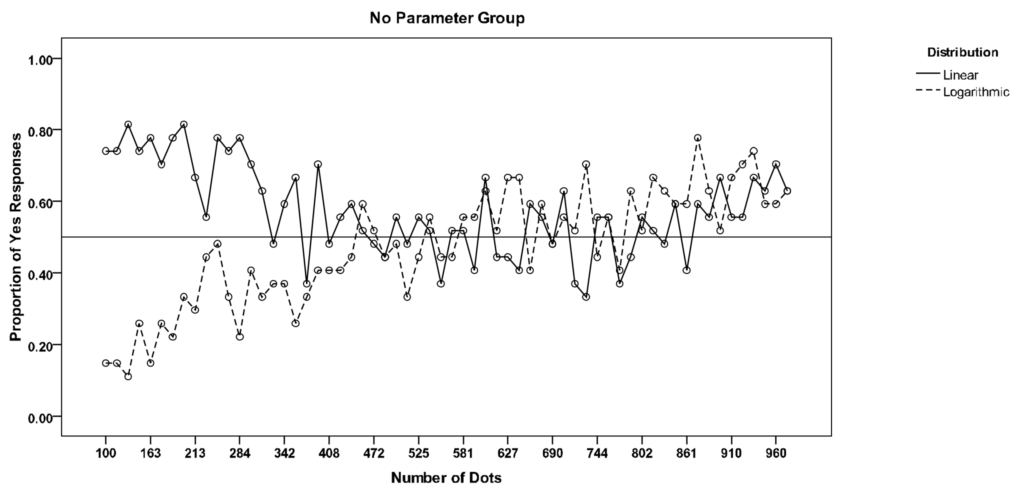


Fig. 8. Parameter and No Parameter proportion of yes responses.

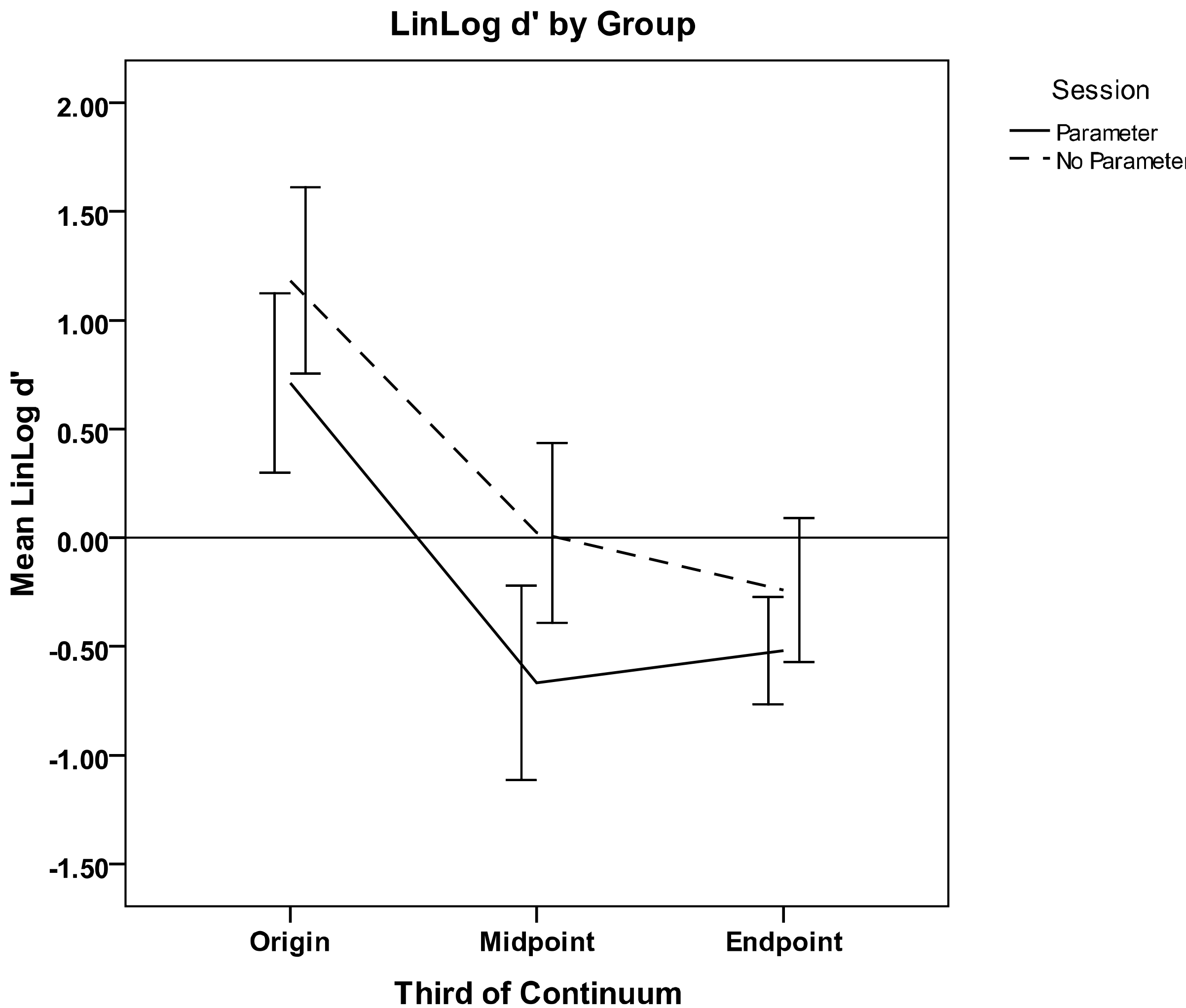


Fig. 9. Parameter and No Parameter LinLog *d'*s along the thirds.

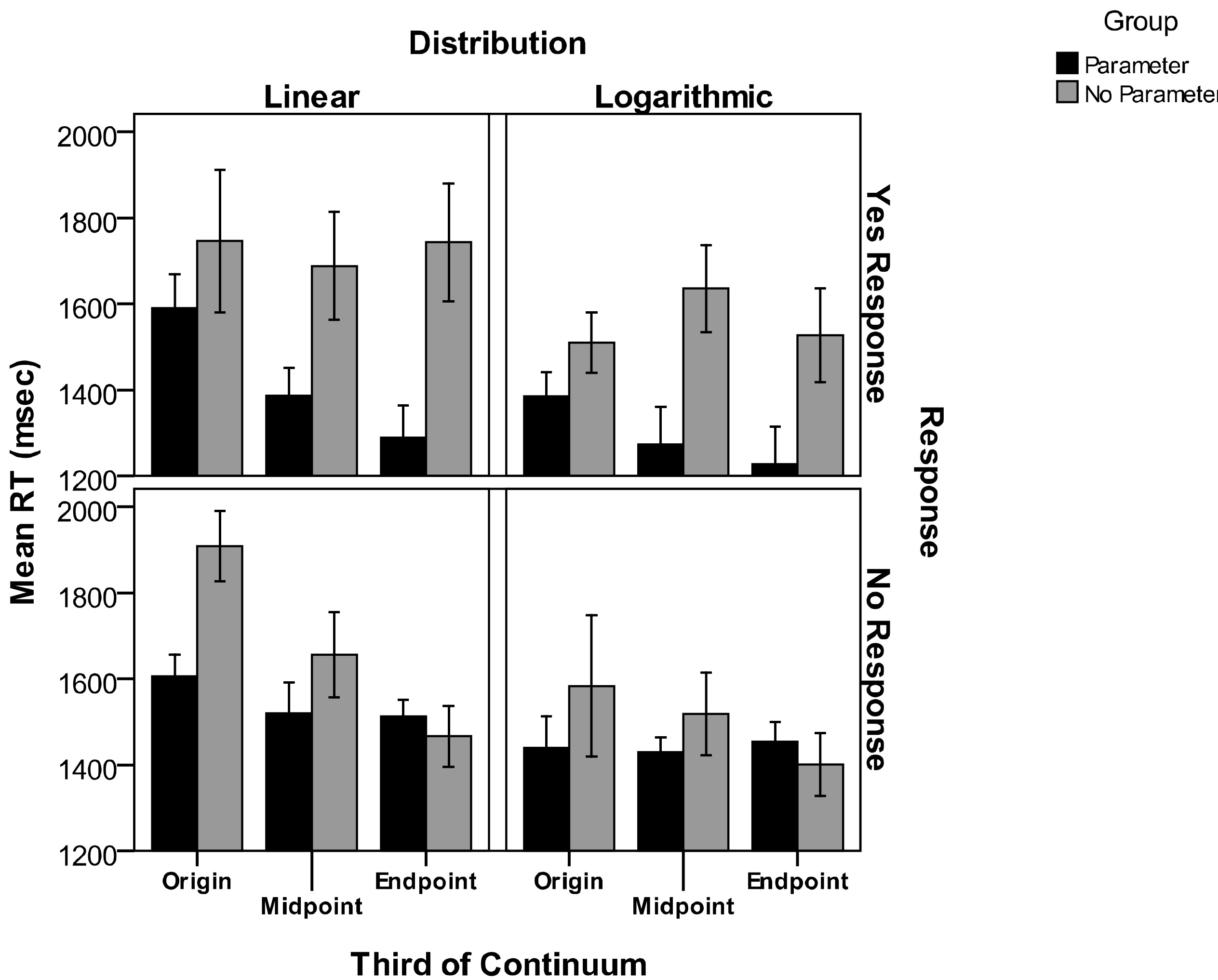


Fig. 10. Mean reaction time along the thirds by distribution and response split by group.

APPENDIX 2

OPRS APPROVAL

UNLV
UNIVERSITY OF NEVADA LAS VEGAS

UNLV IRB Approved JUL 0 6 2010 Expires CE

Social/Behavioral IRB – Exempt Review
Deemed Exempt

DATE: July 9, 2010

TO: **Dr. Mark Ashcraft,** Psychology

FROM: Office of Research Integrity – Human Subjects

RE: Notification of IRB Action by Dr. Ramona Denby Brinson, Chair RDB/CE
Protocol Title: Investigating Adult Number Sense
Protocol # 1005-3475

This memorandum is notification that the project referenced above has been reviewed by the UNLV Social/Behavioral Institutional Review Board (IRB) as indicated in Federal regulatory statutes 45CFR46.

PLEASE NOTE:
Attached to this approval notice is the **official Informed Consent/Assent (IC/A) Form** for this study. The IC/A contains an official approval stamp. Only copies of this official IC/A form may be used when obtaining consent. Please keep the original for your records.

The protocol has been reviewed and deemed exempt from IRB review. It is not in need of further review or approval by the IRB.

Any changes to the exempt protocol may cause this project to require a different level of IRB review. Should any changes need to be made, please submit a **Modification Form**.

If you have questions or require any assistance, please contact the Office of Research Integrity - Human Subjects at IRB@unlv.edu or call 895-2794.

Office of Research Integrity – Human Subjects
4505 Maryland Parkway • Box 451047 • Las Vegas, Nevada 89154-1047

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